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论文题目： Bargaining in the Shadow of the Scraper: Precedent as a
Public Good in AI Training Data Licensing

Bargaining in the Shadow of the Scraper:

Precedent as a Public Good in AI Training Data Licensing

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Abstract

An AI firm can license publishers' content or scrape without authorization and treat expected litigation as a cost. The four-factor fair-use test under 17 U.S.C. § 107 has not produced a bright-line rule for model training, so the value of scraping changes with legal precedent. Collective action, illustrated by Really Simple Licensing (RSL) launched in 2025, is a third path. Existing work on coalition stability does not let members' legal spending change the counterparty's walk-away payoff; bargaining theory treats that outside option as exogenous; public-goods theory stops before bargaining. This paper connects the three. A publisher coalition bargains with the firm under alternating offers, while the firm can quit and scrape. Litigation investment produces a legal-certainty index that raises the firm's expected unauthorized-use cost and spills over to outsiders. Every coalition size is tested for internal-external (IS/ES) stability and robustness to coordinated exit. At the baseline scale, the grand coalition is stable and robust. At larger scales, the IS/ES-stable coalition size is interior and vulnerable to coordinated deviation. Several medium-sized coalitions can have higher per-member payoffs than one grand coalition, consistent with observed collective-licensing markets, but this does not guarantee stability.

Keywords: collective licensing, coalition stability, outside option, litigation investment, legal-precedent spillover, fair use

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1. Introduction

An AI firm has two main ways to obtain publishers' content. It can negotiate a license, or it can use the content without authorization and treat possible litigation as a cost. Whether training a large language model on copyrighted material qualifies as fair use is assessed under the four-factor test in 17 U.S. Code § 107 (2018). However, no bright-line rule has yet been established. A third possibility is collective licensing, such as Really Simple Licensing (RSL), which launched in September 2025.

Existing research on coalition stability does not explain what happens when legal spending changes the outside option. Bargaining theory usually treats that option as exogenous, while public-good theory generally stops before bargaining begins. This gap leads to the research question: How does the legal precedent created through litigation investment affect the stable size of a collective licensing coalition?

This paper brings these strands of research together. A coalition of publishers bargains with the AI firm through alternating offers, while the firm retains the outside option of scraping the publishers' content. The coalition can invest in litigation to increase legal certainty. That investment raises the firm's cost of scraping and creates spillover benefits for publishers outside the coalition. The analysis tests each coalition size for internal and external (IS/ES) stability, and then applies a stronger test that allows members to exit as a coordinated group.

At the baseline scale, the grand coalition is both stable and robust to coordinated exit. At a larger scale, however, the IS/ES-stable coalition is smaller than the full population and remains vulnerable to coordinated deviation. In addition, several medium-sized coalitions may provide

higher net payoffs per member than a single coalition containing the same publishers. However, higher payoffs alone do not guarantee stability.

Sections 2 and 3 provide legal background and relevant literature. Sections 4–5 build the model and solve the equilibrium. Section 6 extends to multiple coalitions and heterogeneity.

2. Background

The AI firm's choice between licensing and unauthorized use depends on the legal environment. Scraping remains a valuable outside option because copyright law has not yet established a clear rule on whether model training is fair use. Its payoff is uncertain because a new settlement or ruling can make scraping more or less attractive.

Publishers therefore face choices. They can negotiate, sue, or pool their rights through collective licensing. They also choose how much to spend on litigation, even though the resulting legal change may benefit publishers that did not contribute. These choices link legal investment, bargaining, and coalition formation.

This section documents the legal uncertainty, bargaining and litigation strategies, and collective licensing.

2.1 Legal Uncertainty and the Scraper Outside Option

The four-factor test in 17 U.S. Code § 107 (2018) evaluates whether training a large language model on copyrighted material qualifies as fair use, and no bright-line rule is settled. As of 2026, the question remains with district courts, which have resolved only pieces of the problem for particular works. Two U.S. cases show that unauthorized training remains a practical but legally unstable outside option.

The first is *Bartz v. Anthropic*. The work in question was books. The alleged actions include training Claude on copies of those books and downloading millions of works from shadow libraries as digital collections. In June 2025, Judge William Alsup granted summary judgment and ruled that the use of legally obtained books for training is fair use, but assembling and keeping the pirated library was not. The same use has different expected liability depending on how the data is obtained.

The second is *The New York Times Company v. Microsoft Corporation* and OpenAI. The works at issue are news articles. The alleged conduct is unauthorized copying of articles for model training and generation of outputs that substitute for the originals. This lawsuit was filed in December 2023, and as of September 2026, the case remains pending in the Southern District of New York. The delay itself carries an economic cost: a rights holder that refuses to negotiate a license cannot promptly secure a judgment for definite damages either.

These cases have two implications for the market in this paper. First, unauthorized training is neither free nor foreclosed by a settled rule, so its expected value remains an outside option that moves with the legal environment. Second, a published order changes the threats to other publishers and AI firms. Later sections therefore treat legal investments as spillovers, not purely private costs.

2.2 Licensing, Litigation, and Collective Licensing

The negotiation process between publishers and AI companies can be classified into three types: direct bargaining, bargaining-litigation, and direct litigation. *Bartz v. Anthropic* is direct litigation without bargaining, while *NYT v. Microsoft* and *OpenAI* is the path of bargaining-

litigation.

A third path is collective action. In September 2025, publishers including Yahoo, Reddit, and Medium launched Really Simple Licensing (RSL), administered by the RSL Collective. RSL provides machine-readable terms for AI training on the model of ASCAP and BML. RSL participation is voluntary, and enforcement is untested. The question is whether the coalition can raise its bargaining weight, not only reallocating surplus.

The model is built to match this mechanism. An AI firm bargains with a coalition or a single publisher and can walk away to unauthorized training. Litigation changes the costs of walking away, but coalition size is considered separately.

3. Literature Review

The section unfolds from coalition stability, bargaining with outside options, and private provision of public goods. Although the literature is developed, none lets right holders spend on a public good that changes the counterparty's walk-away payoff, or lets that surplus feed back into coalition stability.

3.1 Coalition Formation and Stability

The model uses the internal-external stability framework developed by d'Aspremont, Jacquemin, Gabszewicz, and Weymark (1983). A coalition is internally stable when no member wants to leave; it is externally stable when no outsider wants to join. Bloch (1996) studies sequential coalition formation under a fixed rule for dividing gains within a coalition. Ray and Vohra (1999) instead derive that rule as part of the stationary subgame-perfect equilibrium (SPE). However, none of these models allows a coalition's own legal spending to change the

counterparty's payoff from walking away.

This paper keeps IS/ES stability as the baseline and adds coordinated exit as a stronger test. It also considers a larger population of publishers. Collective legal expenditure raises the cost of unauthorized training and changes payoffs. The value of a coalition therefore includes changes in the legal environment if members' litigation makes scraping more expensive. The standard IS/ES stability test misses free-riding on legal spillovers, and a group may leave together.

3.2 Bargaining Theory and Outside Options

Rubinstein (1982) derives the unique SPE division of surplus in alternating-offers bargaining. Binmore, Rubinstein, and Wolinsky (1986) show that the continuous-time limit yields the Nash solution. Shaked and Sutton (1984) introduce an outside option for a player who can switch partners. This option becomes binding only when its payoff exceeds what the player could obtain by delaying agreement at the bargaining table.

Applied studies generally treat the outside option as exogenous. Here, it depends on the coalition's own legal investment, while the bargaining weight remains unchanged. Once that weight is set, the outside option places a cap on the payoff the coalition can obtain.

3.3 Public Goods, Joint Production, and Spillovers

Cornes and Sandler (1984) introduce joint production, in which a private input generates both private and public effects. Bergstrom, Blume, and Varian (1986) establish the existence and uniqueness of Nash provision. Olson (1965) shows that free-riding becomes more severe as a group grows.

In this literature, the public good is usually valued for its own sake, and its provision is

analyzed separately from its later use. In the present setting, publishers invest in legal certainty because it improves the subsequent bargain. Outsiders can benefit from that certainty without joining the coalition. The coalition's advantage therefore depends on how it brings members together and coordinates their collective contribution.

3.4 What This Paper Adds

Taken together, these strands leave an important connection unexplored. Coalition-stability models do not allow legal investment to change the counterparty's outside option, limiting their ability to explain stability when that option changes. Bargaining theory usually treats the option as fixed, so it does not explain how free-riding on legal investment feeds back into licensing payoffs. Public-good models, in turn, stop before the bargain begins.

This paper connects these mechanisms through litigation investment and the legal certainty it produces as a jointly produced public good. The resulting spillovers raise the AI firm's cost of training without a license, reducing its outside-option payoff. Under the outside-option principle, this payoff limits what the coalition can obtain; the bargaining power itself remains unchanged.

Publishers then use the resulting member and outsider payoffs to decide whether to join, stay, leave, or exit together. The analysis asks when collective licensing is IS/ES stable and when a coalition of that size is vulnerable to coordinated exit. It also examines when, at a larger scale, medium-sized coalitions can form a stable structure in place of a grand coalition.

4. Model

4.1 Players and Information

There are n publishers, indexed $i = 1, \dots, n$, and one AI firm. Publishers are ex ante

symmetric. All players have complete information. The analysis below considers one bargaining episode at a time.

4.2 Timing

A coalition $S \subseteq N$ is a set of publishers who (i) bargain with the AI firm as a single unified party and receive one joint payoff ϕ_S that is split among members, and (ii) jointly decide their litigation-investment strategy. A coalition does not control the AI firm's behavior, nor the litigation decisions of publishers outside S .

Coalition formation is not modeled as a full non-cooperative game, such as an open-membership sequential game with acceptance or rejection at every step. The paper evaluates every possible size $s = |S|$ for self-consistency using the internal- and external-stability tests.

The payoff of a coalition depends on how the complementary players, which are publishers outside S , organize themselves. Outsiders may free ride on the litigation precedent or compete with S for a share of the AI firm's fixed licensing budget.

Θ denotes a legal certainty index, which is the linear aggregate of individual publishers' litigation investment l_i .

Assumption 1. If the AI firm chooses to “quit and scrape” during any round of bargaining, the current bargaining episode ends permanently. The AI firm cannot return to the table later within the same episode.

Assumption 2. The accumulation of Θ happens before negotiation, functioning as a condition variable.

Assumption 3. The time interval between negotiation rounds approaches zero, since the

bargaining process is significantly shorter than the time needed for substantial changes to occur in the legal proceedings. While the former process takes place within days to weeks, the latter requires months to years.

Assumption 4. Both sides have complete information.

4.3 Technologies

4.3.1 Surplus

Total surplus $V(\text{data})$ does not vary with Θ . When a coalition S is a bargaining unit, $V(\text{data})$ is replaced by size-dependent $V_{total}(s) = A \cdot s^{\frac{1}{\sigma}}$, where σ represents the elasticity of substitution across publishers' data, and A is a budget-scale parameter. An independent publisher facing a coalition of size s bargains over $V_{marginal}(s) = \frac{A}{\sigma} \cdot s^{\frac{1}{\sigma}-1}$.

AI firm's true outside option: $\pi_{scrape}(\Theta) = V(\text{data}) - c^*(\Theta)$, where $c^*(\Theta)$ is the AI firm's minimized expected cost from unauthorized use. Payoffs are measured as incremental returns from AI licensing. Publishers' disagreement payoffs are assumed to be zero.

4.3.2 The AI Firm's Avoidance-cost Technology

Let $q(e, \Theta) = \Theta(1 - e)$ denote the AI firm's residual expected liability intensity after choosing avoidance effort e , $D(\Theta) = D_0$ a constant baseline damage rate, and $c(e) = \frac{k}{2}e^2$ the convex cost of avoidance effort. The AI firm solves:

$$\min_{e \in [0,1]} D_0 \Theta(1 - e) + \frac{k}{2}e^2$$

The solution is the truncated stationary point:

$$e^*(\Theta) = \min \left\{ \frac{D_0 \Theta}{k}, 1 \right\}$$

And substituting back:

$$c^*(\Theta) = \begin{cases} D_0\Theta - \frac{D_0^2}{2k}\Theta^2 & \text{if } \Theta \leq \frac{k}{D_0} \\ \frac{k}{2} & \text{if } \Theta > \frac{k}{D_0} \end{cases}$$

The function is strictly increasing and concave on $\Theta \leq \frac{k}{D_0}$, and is constant after. Therefore, it is globally concave.

The AI firm's realized outside option $\pi_{scrape}(\Theta) = V(data) - c^*(\Theta)$ depends on Θ and need not equal its Rubinstein share of the surplus. Whether this outside option constrains the bargaining outcome, and if so, how is established formally below.

4.4 Bargaining

4.4.1 Bargaining Weight $\beta(S)$

As $\Delta t \rightarrow 0$, $\delta_{AI}, \delta_{pub} \rightarrow 1$, and the first-mover advantage disappears.

The equilibrium share converges to $\beta(S) = \frac{r_{AI}}{r_{pub(S)} + r_{AI}}$, where

$$\delta_i(\Delta t) = e^{-r_i \Delta t}$$

Thus,

$$r_i = -\frac{\ln \delta_i(\Delta t)}{\Delta t}$$

is the continuous-time discount rate.

For $\beta(S)$ to increase in $s=|S|$, $r_{pub}(S)$ must decrease in s . Maintaining a bargaining coalition imposes a burden on the coalition's collective patience per period. The burden is split among s members, with per-member burden intensity decreasing in s . Therefore, this paper introduces f , a coalition-maintenance burden-intensity parameter. With individual baseline discount rate r_0 :

$$r_{pub} = r_0 + f/s$$

which gives a closed form:

$$\beta(S) = \frac{r_{AI}}{r_0 + \frac{f}{S} + r_{AI}}$$

$\beta(S)$ is strictly increasing with s . It attains its minimum $\frac{r_{AI}}{r_0 + f + r_{AI}}$ at $s=1$, and an upper bound $\frac{r_{AI}}{r_0 + r_{AI}}$ as s approaches infinity.

4.4.2 Protocol and Two Disagreement Payoffs

The coalition and the AI firm bargain by alternating offers with discount rates r_{pub} and r_{AI} . If one side rejects an offer, the game continues, and both sides have zero payoff during the delay. The theoretical disagreement point of the bargaining $(d_{pub}, d_{AI}) = (0,0)$ occurs when they fail to reach an agreement.

The AI firm can quit and scrape at any decision node, receiving a one-time payoff $\pi_{scrape}(\Theta) = V - c^*(\Theta)$.

4.4.3 Outside Option Principle

Proposition 1. Consider the following game: the bargaining unit and the AI firm bargain via alternating offers at discount factors δ_{pub} , δ_{AI} . At any decision point of the AI firm, the AI firm may quit and scrape, receiving $\pi_{scrape}(\Theta)$ as a one-time, immediately realized payoff. The episode terminates, and the AI firm cannot return to bargaining. In the continuous-time limit, the bargaining unit's equilibrium payoff is:

$$\phi_{pub}(\Theta, S) = \min\{c^*(\Theta), \beta(S) \cdot V(\text{data})\}$$

for any realized value of Θ (proof in Appendix A.2).

The kink $\Theta^*(S)$ solves $c^*(\Theta) = \beta(S)V(\text{data})$:

$$\Theta^*(S) = \left(\frac{k}{D_0}\right) \cdot \left[1 - \sqrt{1 - \frac{2\beta(S)V(\text{data})}{k}}\right] \quad (\text{exists iff } k \geq 2\beta(S)V(\text{data}))$$

Interpretation. The condition $k \geq 2\beta(S)V(\text{data})$ compares the ceiling of avoidance costs $k/2$, with the coalition's unconstrained Rubinstein share. If it holds, a finite $\Theta^*(S)$ exists: litigation can raise $c^*(\Theta)$ up to $\beta(S)V(\text{data})$, after which the outside option ceases to bind and further spending does not change the split. If not, $c^*(\Theta) \leq k/2 < \beta(S)V(\text{data})$, so scraping always constrains bargaining. Thus, larger s favors the second regime, while higher k favors the first.

Proposition 2. $\phi_{pub}(\Theta, S)$ is globally concave and nondecreasing in Θ . If $\beta(S)V(\text{data}) \leq k/2$, it is strictly increasing for $0 \leq \Theta < \Theta^*(S)$ and constant at $\beta(S)V(\text{data})$ thereafter. If $\beta(S)V(\text{data}) > k/2$, no finite $\Theta^*(S)$ exists. The payoff is strictly increasing for $0 \leq \Theta < k/D_0$ and constant at $k/2$ thereafter.

4.5 The Litigation Investment Problem

Let a share $\rho \in [0,1]$ of legal certainty generated by publisher i 's litigation investment l_i spill over as a public precedent shared by everyone, and $1 - \rho$ remains as i 's own leverage.

Define the effective legal certainty index relevant to S coalition bargaining:

$$\Theta_S^{\text{eff}} = \rho \cdot \sum_{j \in N} l_j + (1 - \rho) \cdot \sum_{i \in S} l_i$$

$\rho = 1$ corresponds to “pure public good”, while $\rho = 0$ to the opposite.

For a coalition S of size s and the complementary set of independent publishers, the effective

legal-certainty indices relevant to each side's own bargaining are

$$\begin{aligned}\Theta_S^{\text{eff}} &= \rho \cdot \sum_{j \in N} l_j + (1 - \rho) \cdot \sum_{i \in S} l_i = s \cdot l_S + \rho(n - s) \cdot l_O \\ \Theta_O^{\text{eff}} &= \rho \cdot \sum_{j \in N} l_j + (1 - \rho) \cdot l_i = \rho s \cdot l_S + [\rho(n - s) + (1 - \rho)] \cdot l_O\end{aligned}$$

where l_S, l_O denote the investment levels of a coalition member and an independent publisher, respectively.

Coalition S bargains and invests as a unified party. Publishers outside S remain individually non-cooperative. The resulting construction is:

$$w_Y(S) = \phi_S(\Theta_S^{\text{eff}*}(S), S) - \frac{\gamma}{2} \sum_{i \in S} [l_i^*(S)]^2$$

Here, $w_Y(S)$ denotes the coalition's total net payoff, evaluated at the litigation-investment equilibrium induced by coalition S , with all publishers outside S acting as independent singletons.

Lemma 1. For any individual i , regardless of ρ , the marginal contribution of l_i to the effective legal-certainty index of i 's own bargaining unit is $\frac{\partial \Theta_{\text{eff}^{\text{group}(i)}}}{\partial l_i} = \rho + (1 - \rho) = 1$.

An independent publisher j chooses l_j to maximize $\pi_j(l_j; l_{-j}) = \phi_j(\Theta_{\text{eff}^{\{j\}}}) - \frac{\gamma}{2} l_j^2$, where $\phi_j(\Theta) = \min\{c^*(\Theta), \beta(1) \cdot V_{\text{marginal}}\}$

Proposition 3. The optimal individual litigation investment for independent publishers under the individual Nash equilibrium satisfies

$$l_O^* = \min\{l_O^{\text{FOC}}, l_O^{\text{kink}}\}, \quad \gamma l_O^{\text{FOC}} = c^{*'}(\Theta_O^{\text{eff}})$$

The optimal investment for coalition members solving a joint maximization problem

satisfies

$$l_s^* = \min\{l_s^{\text{FOC}}, l_s^{\text{kin}}\}, \quad \gamma l_s^{\text{FOC}} = c^{*'}(\theta_s^{\text{eff}})$$

Here, l_g^{FOC} , for $g \in \{O, S\}$, denotes the nonnegative investment maximizing the uncapped objective based on $c^*(\theta_g^{\text{eff}})$. The displayed first-order equality applies only when $l_g^{\text{FOC}} > 0$. Otherwise, $l_g^{\text{FOC}} = 0$. Define l_g^{kin} as the smallest nonnegative own investment that reaches the bargaining cap, holding other decision-makers' investments fixed. Set $l_g^{\text{kin}} = 0$ if spillovers alone already reach the cap, and $l_g^{\text{kin}} = +\infty$ if the cap exceeds $k/2$.

(proofs in Appendix B)

The two conditions share an identical functional structure given by $\gamma l = c^{*'}(\theta)$, which contains no scale dilution driven by coalition size. A coalition member θ_s^{eff} accumulates at rate s , while an independent publisher contributes its own l_0 . Collective decision-making within coalitions consolidates s separate private investment choices into a unified decision rule.

4.6 IS/ES Stability

Definition 1. A coalition size s is IS/ES-stable if no incumbent member gains by leaving and no outsider gains by joining, adapting d'Aspremont, Jacquemin, Gabszewicz & Weymark's (1983) internal-plus-external-stability framework for cartel size. Writing $\pi_{\text{member}}(s)$ for a coalition member's payoff when the coalition has size s , and $\pi_{\text{out}}(s)$ for an independent publisher's payoff when the coalition has size s :

Internal Stability (IS): no member wants to leave

$$\pi_{\text{member}}(s) \geq \pi_{\text{out}}(s - 1)$$

External Stability (ES): no outsider wants to join

$$\pi_{out}(s) \geq \pi_{member}(s + 1)$$

At $s = 1$, internal stability is vacuous; at $s = n$, external stability is vacuous.

Definition 2. An IS/ES-stable coalition size s is coalitional-deviation robust if no group of $m \geq 2$ incumbent members can jointly gain by leaving together, for every feasible $m = 2, \dots, s - 1$:

$$\pi_{member}(s) \geq \pi_{out}(s - m)$$

Coalitional-deviation robustness is strictly stronger than IS/ES stability, since the latter only rules out single-member deviations.

5. Equilibrium

This section applies the rules of Section 4 in the order they unfold. A coalition of size s forms. Given that size, the coalition and independent publishers choose to invest in litigation, determining the legal-certainty indices that will govern bargaining. The resulting payoffs are realized through the bargaining protocol, and the stability of the coalition size itself is then assessed.

5.1 Outcome Given Coalition Size

Equilibrium Investment: For each candidate coalition size s , the first-order conditions of Proposition 3 form a two-equation nonlinear system in (l_s, l_o) , each equation truncated by its respective kink. The resulting equilibrium legal-certainty indices are

$$\Theta_s^{\text{eff}}(s) = s \cdot l_s^*(s) + \rho(n - s) \cdot l_o^*(s)$$

$$\Theta_o^{\text{eff}}(s) = \rho s \cdot l_s^*(s) + [\rho(n - s) + (1 - \rho)] \cdot l_o^*(s)$$

Substituting $\Theta_s^{\text{eff}}(s), \Theta_o^{\text{eff}}(s)$ into Proposition 1, together with the size-dependent surplus

terms $V_{\text{total}}(s) = A \cdot s^{1/\sigma}$ and $V_{\text{marginal}}(s) = (A/\sigma) \cdot s^{(1/\sigma)-1}$, gives each type's equilibrium payoff as a function of coalition size:

$$\pi_{\text{member}}(s) = \frac{1}{s} \min \left\{ c^* \left(\Theta_s^{\text{eff}}(s) \right), \beta(s) A s^{1/\sigma} \right\} - \frac{\gamma}{2} [l_s^*(s)]^2$$

$$\pi_{\text{out}}(s) = \min \left\{ c^* \left(\Theta_o^{\text{eff}}(s) \right), \beta(1) \frac{A}{\sigma} s^{1/\sigma-1} \right\} - \frac{\gamma}{2} [l_o^*(s)]^2$$

Numerical illustration. Table 1 reports the equilibrium for $n = 20, r_{AI} = 0.15, r_0 = 0.05, f = 3.0, D_0 = 1.0, k = 8.0, A = 3.2, \sigma = 1.3, \gamma = 1.6, \rho = 0.5$.

Table 1

Equilibrium Investment and Payoffs by Coalition Size ($\rho=0.5$)

s	$\beta(S)$	l_s^*	l_o^*	Θ_s^{eff}	Θ_o^{eff}	$\pi_{\text{member}}(s)$	$\pi_{\text{out}}(s)$
1	0.0469	0.0781	0.0077	0.1514	0.1162	0.1451	0.1153
2	0.0882	0.2483	0.0000	0.4966	0.2483	0.1913	0.0983
4	0.1579	0.4087	0.0000	1.6347	0.8174	0.2333	0.0838
5	0.1875	0.4494	0.0000	2.2472	1.1236	0.2247	0.0796
10	0.3000	0.3509	0.0000	3.5088	1.7544	0.1754	0.0678
19	0.4191	0.2516	0.0000	4.7799	2.3899	0.1258	0.0585

Note. Other values of s in the range of 1 to 19 are omitted. $\pi_{\text{member}}(s)$ peaks at s=4, while $\pi_{\text{out}}(s)$ is monotonically decreasing throughout. (full table, Appendix C, Table C1).

5.2 Two Externalities

Channel A: a negative externality with no ceiling. Suppose the AI firm's total licensing budget is $V_{\text{total}}(x) = A \cdot x^{\frac{1}{\sigma}}$ ($\sigma > 1$). Once a coalition of size s signs first, an independent publisher's marginal share of the remaining budget, $V_{\text{marginal}}(s) = \frac{A}{\sigma} \cdot s^{\frac{1}{\sigma}-1}$, is strictly decreasing in s.

Channel B: a positive externality, but with a ceiling. $\Theta^*(S)$ is increasing in $\beta(S)$

(Proposition 1), so a larger coalition creates a stronger incentive to invest in litigation, and a fraction ρ of that investment spills over to benefit outsiders as public precedent. When an outsider's own saturation is reached, further coalition growth brings no additional benefit. The net effect of coalition size on an outsider's payoff is therefore ambiguous.

5.3 Stability of Coalition Size

A coalition of publishers S , with $|S|=s$, forms and acts as the unified bargaining and investment unit. The remaining $n-s$ publishers act individually. The analysis evaluates every candidate size $s = 1, \dots, n$ against the IS/ES-stability criterion of Definition 1, using the equilibrium payoffs derived in Sections 5.1.

For $s = 2, \dots, n$, define the entry margin

$$\Delta(s) = \pi_{\text{member}}(s) - \pi_{\text{out}}(s-1)$$

This entry margin compares the two net payoffs from joining a coalition of size $s-1$ (becoming its s -th member) rather than remaining independent when the coalition has size $s-1$.

For an interior coalition size $2 \leq s \leq n-1$, internal and external stability conditions are equivalent to

$$\text{IS}(s) \Leftrightarrow \Delta(s) \geq 0$$

$$\text{ES}(s) \Leftrightarrow \pi_{\text{out}}(s) \geq \pi_{\text{member}}(s+1) \Leftrightarrow \Delta(s+1) \leq 0$$

Hence, an interior size $2 \leq s^* \leq n-1$ is IS/ES-stable if and only if

$$\Delta(s^*) \geq 0, \quad \Delta(s^* + 1) \leq 0$$

This section first explains the behavior of Δ (Proposition 4), then provides a general

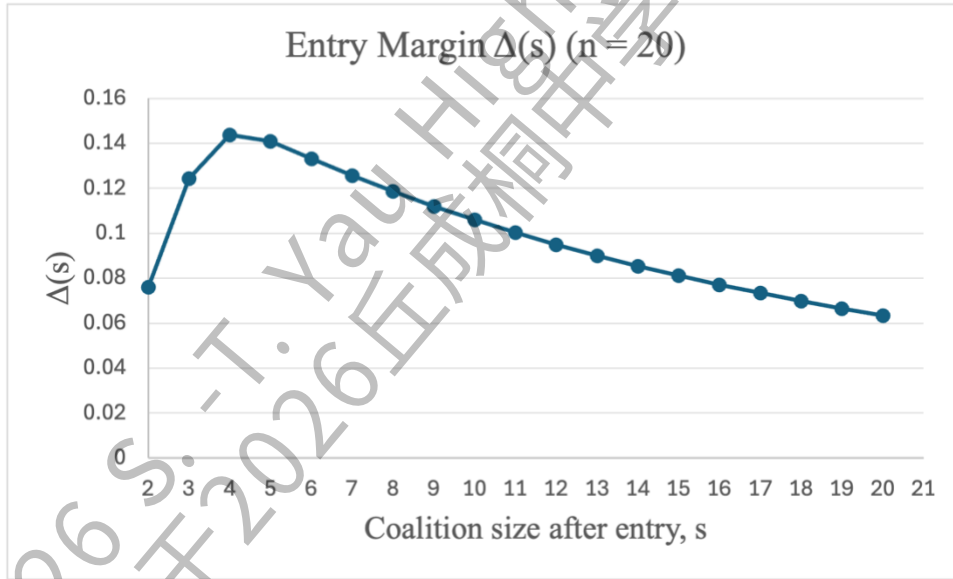
existence-and-uniqueness theorem formulated in terms of Δ (Proposition 5), and finally verifies both the mechanism and the general condition of the calibration.

Proposition 4. Define $\Delta(s) = [\text{Channel B's marginal contribution to } \pi_{\text{out}} \text{ at } s] - [\text{Channel A's marginal harm to } \pi_{\text{out}} \text{ at } s]$, the marginal net effect of coalition size on an outsider's payoff.

For small s , Channel B's public-precedent effect can dominate, and $\pi_{\text{out}}(s)$ can increase with s . Once s is large enough that the equilibrium θ_0^{eff} reaches an outsider's own saturation threshold, Channel B's marginal contribution vanishes, and $\pi_{\text{out}}(s)$ is governed by Channel A, which is strictly decreasing.

Figure 1

Entry Margin $\Delta(s)$



Note. $\Delta(s)$ stays positive throughout (peak at $s = 4$), confirming the boundary stable size $s^* = 20$ in Proposition 6.

Proposition 5. Suppose the entry margin $\Delta(s)$ is non-increasing in s over the domain $s =$

$2, \dots, n$. An IS/ES-stable coalition size s^* exists. If $\Delta(s)$ is strictly decreasing and $\Delta(s) \neq 0$ for every $s = 2, \dots, n$, the IS/ES-stable coalition size is unique. If $\Delta(s)$ is strictly decreasing but equals zero at an integer j , the two adjacent sizes $j - 1$ and j are both stable under the weak IS/ES inequalities.

In particular, if $\Delta(s) > 0$ for every $s = 2, \dots, n$, the stable size is the boundary $s^* = n$. If $\Delta(s) < 0$ throughout this domain, the stable size is the boundary $s^* = 1$. Full values underlying Figure 1 are in Appendix C.2, Table C2.

Proposition 6. Under the calibration of Table 1, with $n = 20$ and entry margins satisfy

$$\Delta(s) > 0 \quad \text{for every } s = 2, \dots, 20$$

Consequently, the grand coalition $s^* = 20$ is the unique IS/ES-stable coalition size. Under a sequential adjustment rule where one outsider joins whenever joining strictly increases its own net payoff, every initial size $s = 1, \dots, 19$ reaches $s = 20$.

Figure 1 shows that the entry margin peaks at $s = 4$, with $\Delta(4) = 0.143764832$, and then decreases. It nevertheless remains positive through the final entry, with $\Delta(20) = 0.063465354$. Thus, the gain from joining becomes smaller as the coalition grows beyond four members, but joining remains profitable even for the last outsider.

Proposition 7. The IS/ES-stable coalition size $s^* = 20$ is also coalitional-deviation robust to coordinated exit by any group of $m = 1, \dots, 19$ members.

All values fall short of $\pi_{\text{member}}(20) = 0.1220$, and a complete check over $m = 1, \dots, 19$ confirms this holds throughout. No coordinated exit is ever profitable. Appendix C.3 and Table C3 provide the complete check.

Notably, the ordinary operation of Channel A is that $\pi_{out}(20 - m)$ increases as m increases and the coalition contracts. However, even at $m=19$, where the coalition is reduced to a single member and the defectors obtain the highest outsider payoff, $\pi_{out}(1) = 0.1154$, this still falls short of $\pi_{member}(20) = 0.1220$.

Section 5.4 shows that coordinated deviation does become beneficial once the margin between $\pi_{member}(s)$ and $\pi_{out}(s)$ disappears close to an interior crossing point, and the recursive issues this paragraph avoids become relevant again.

5.4 Coalition Size at Scale

Propositions 6 and 7 establish the grand-coalition outcome under the calibration of Table 1 ($n=20$). However, both results depend on the specific calibration feature, and real-world publishers would not be confined to this scale. As of June 2026, SPUR brought together 36 publishers and affiliate organizations (Association of Online Publishers, 2026), while the RSL Collective had about 1,500 media members at the time of reporting (Adami, 2026). It remains unclear whether the grand-coalition outcome persists as n grows, or it breaks down once the coalition reaches a certain scale. This subsection derives an asymptotic answer and shows its limitation on coalitional deviations. A full characterization of IS/ES-stable coalition structures at scale is left to future work.

Proposition 8. Suppose $\sigma > 1$ and $\rho > 0$. As the population grows, consider coalition sizes $s < n$ for which $l_0^*(s) = 0$ at sufficiently large s . The effective indices are then $\Theta_s^{\text{eff}}(s) = sl_s^*(s)$ and $\Theta_0^{\text{eff}}(s) = \rho\Theta_s^{\text{eff}}(s)$, independent of n . For sufficiently large s , the coalition is on the interior investment branch and

$$\pi_{\text{member}}(s) = \frac{kD_0^2}{2(\gamma k + D_0^2 s)} = \frac{k}{2s} - \frac{\gamma k^2}{2D_0^2 s^2} + O(s^{-3})$$

while

$$\pi_{\text{out}}(s) \sim \beta(1) \frac{A}{\sigma} s^{1/\sigma-1}.$$

Consequently, $\Delta(s) < 0$ for sufficiently large s . Equating the leading-order terms gives the approximate scale

$$s_{\text{cross}} \approx \left[\frac{k/2}{\beta(1)A/\sigma} \right]^\sigma$$

This approximation does not determine the exact stable integer size. The proof is provided in Appendix D.1.

Remark. For the calibration of Table 1, the leading-order scale is approximately 100.44. Propositions 6 and 7 are established by the exact finite-population checks, independently of this approximation.

Proposition 9. An interior IS/ES-stable size is robust to individual deviation but need not be robust to coalitional deviation, even though it satisfies Definition 1. Under a calibration with large n ($n=200$; other parameters as in Table 1), the interior stable size is $s^* \approx 83$, with $\pi_{\text{member}}(s^*) = 0.0418$. A deviation check shows that single-member exits are unprofitable. Still, deviation of size $m = 2, \dots, 82$ is profitable, where $\pi_{\text{out}}(s^* - m)$ exceeds $\pi_{\text{member}}(s^*)$ for all such m .

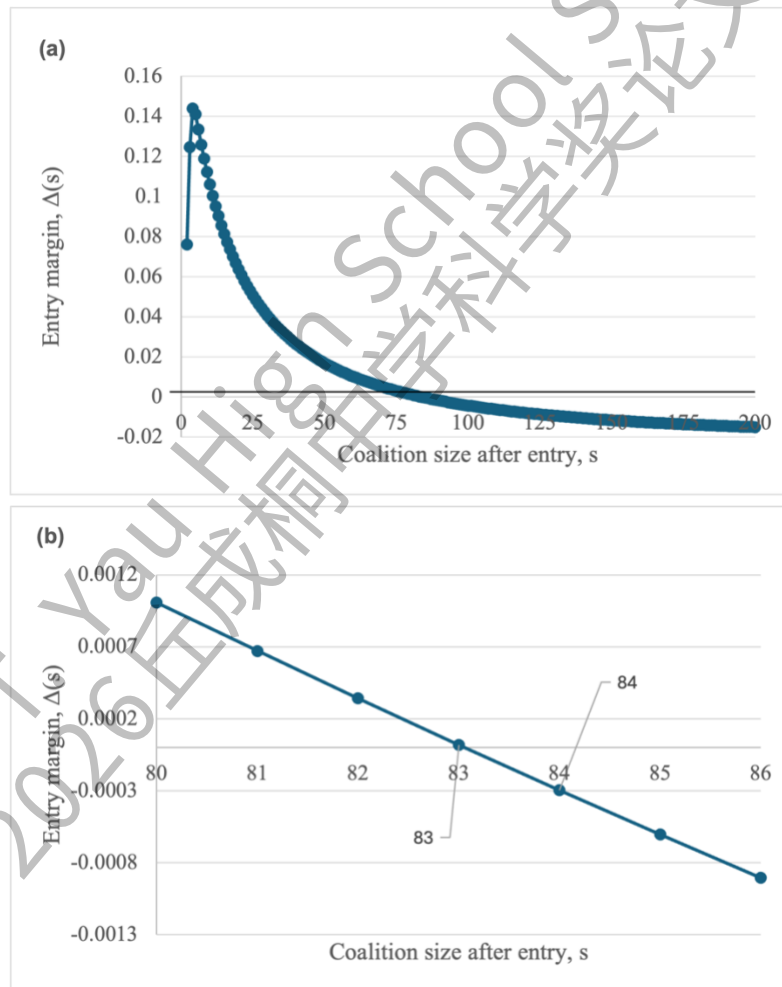
Interpretation. This does not contradict Definition 1 and Proposition 8. It only separates individual-deviation stability from coalitional stability. Proposition 7 avoids this gap at $n = 20$ because $\pi_{\text{member}}(s)$ dominates $\pi_{\text{out}}(s)$ uniformly. That margin disappears by construction

at an internal crossing point ($\Delta(s^*) \approx 0$), resulting in a divergence between the two concepts.

Characterizing IS/ES-stable structures in this regime, possibly involving a partition into several competing coalitions rather than one, is left to future work. As shown in Figure 2, $\Delta(83) = 0.000018832 > 0$ and $\Delta(84) = -0.000295928 < 0$. Together with the full check in Appendix D.2, these inequalities establish $s^* = 83$ as the unique IS/ES-stable coalition size.

Figure 2

Entry Margin and the Interior Stable Coalition Size at $n=200$



Note. Panel (a) reports the entry margin $\Delta(s)$. Panel (b) enlarges the region of sign change between $s=83$ and $s=84$.

6. Extensions

This section develops one extension, provides one robustness implication, and outlines two directions for future work. Section 5.4 (Propositions 8–9) further shows that when the population n is large enough, this single-coalition equilibrium is only individually stable, and groups of $m \geq 2$ members can jointly gain by leaving together.

The observation leads to a question. If a leaving group can form a new coalition rather than singletons, whether the market settles at one dominant coalition or several moderate-size coalitions. This section examines this question for the isolated population of 83 publishers in Proposition 9, then examines cost efficiency with heterogeneous publishers (6.2) and proposes two further extensions for future work (6.3).

6.1 Multiple Coalitions

This subsection allows publishers to form several coalitions. Let

$$\pi = \{S_1, \dots, S_m\}$$

represent a partition of n' publishers into m coalitions with $m \geq 1$, where $s_k = |S_k|$ and $\sum_{k=1}^m s_k = n'$. Each coalition bargains and chooses litigation investment as a unified party. The population n' is considered separately. The extension establishes a mechanism to allocate the AI firm's aggregate valuation $V_{\text{total}}(n')$ across multiple coalitions that bargain simultaneously when $m \geq 2$.

Assumption 5. In the symmetric baseline model, surplus is split equally among a coalition's own members, and litigation investments are chosen jointly to maximize total net payoff.

Across coalitions, the Shapley rule is applied to the characteristic function

$$v(C) = V_{\text{total}} \left(\sum_{k \in C} s_k \right), \quad v(\emptyset) = 0,$$

where C is a subset of coalition blocks (Shapley, 1953). Coalition S_k 's bargaining payment follows

$$\phi_{S_k} = \min\{c^*(\Theta_{S_k}^{\text{eff}}), \beta(s_k) \text{Shapley}_k(\pi)\}.$$

Thus, Shapley values determine the bargaining-payment caps.

Lemma 2. Under Assumption 5, let $l_k^*(\pi)$ denote the equilibrium investment of each member of S_k . Then

$$\Theta_{S_k}^{\text{eff}}(\pi) = s_k l_k^*(\pi) + \rho \sum_{j \neq k} s_j l_j^*(\pi),$$

and the equilibrium per-member net payoff is

$$\pi_{\text{member},k}(\pi) = \frac{1}{s_k} \min\{c^*(\Theta_{S_k}^{\text{eff}}(\pi)), \beta(s_k) \text{Shapley}_k(\pi)\} - \frac{\gamma}{2} [l_k^*(\pi)]^2.$$

Payoffs therefore depend on coalition sizes, Shapley values, and equilibrium litigation spillovers. When the m coalitions have equal size $s = n'/m$,

$$\text{Shapley}_k(\pi) = \frac{V_{\text{total}}(n')}{m}.$$

The proof is provided in Appendix D.3.1.

Definition 3 (Merge/Split Stability, adapting Bloch, 1996; Ray & Vohra, 1999). A partition π is merge/split stable if (i) no individual member gains by moving to an alternative block, (ii) no member of a splitting block gains from any binary split of that block, and (iii) no member of either merging block gains from their merger. The deviation test records whether at least one affected publisher gains, and it does not impose unanimous consent.

Proposition 10. Suppose $\rho \in [0,1)$ and $\beta(s)$ is strictly increasing in s , which holds for

$\beta(s) = r_{AI}/(r_0 + f/s + r_{AI})$ whenever $f > 0$. Then, for the symmetric partition of any population n' into m equal blocks:

- (i) The interior per-member net payoff $\pi_{\text{interior}}(m)$ is strictly increasing in m .
- (ii) As m increases, the transition from the interior investment branch to the Rubinstein-capped branch occurs at most once.
- (iii) If such a transition occurs at an interior point m_c , and the capped net payoff $\pi_{\text{cap}}(m)$ is strictly decreasing over its active range, then $\pi_{\text{member}}(m)$ is single-peaked, with its unique maximum at m_c .

The formulas and proof are provided in Appendix D.3. Under the calibration in Table 1 and the population $s^* = 83$ in Proposition 9, the equal-size formulas evaluated at integer coalition counts $m = 1, \dots, 83$ yield their highest net payoff at

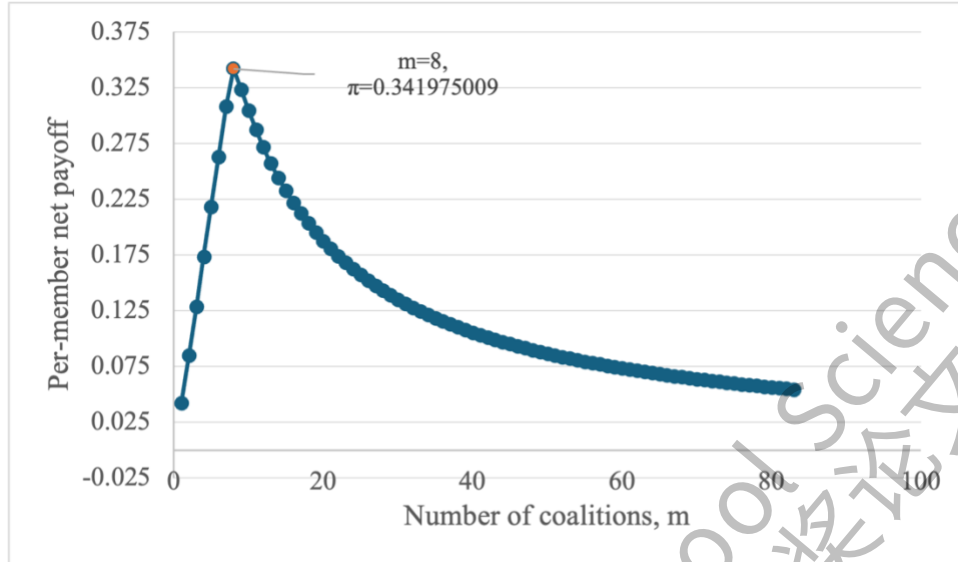
$$m = 8, \quad s = 83/8 = 10.375, \quad \pi_{\text{member}} = 0.341975009.$$

The equal-size approximation in Proposition 10 identifies eight coalitions for the isolated population of 83 publishers. We test the partition $\pi = \{S_1, \dots, S_8\}$, with three 11-member coalitions and five 10-member coalitions.

Figure 3 plots the equal-size payoff comparison for the isolated population of 83 publishers. The per-member net payoff reaches its maximum at $m = 8$, but this payoff maximum alone does not establish stability.

Figure 3

Per-Member Net Payoffs under the Equal-Size Approximation ($n'=83$)



Note. Parameters are as in Table 1. The isolated population of 83 publishers is divided into m equal-sized coalitions, allowing fractional coalition sizes $s = 83/m$. Among integer coalition counts $m = 1, \dots, 83$, the highest per-member net payoff is 0.341975009 at $m = 8$. This payoff comparison does not prove partition stability. Lines connect integer observations.

Proposition 11. Under Assumption 5 and the calibration of Table 1, the 83-publisher partition π admits no profitable individual move to an existing coalition and no profitable merger of two coalitions. It is not merge/split stable under Definition 3. Splitting an 11-member coalition into 1+10 or 2+9 strictly increases the net payoff of members in the larger resulting coalition.

This result follows checks for every individual move, every binary split of one coalition, and every merger of two coalitions. Table 2 summarizes the results. All payoff-distinct comparisons and numerical verification are provided in Appendix D.4.

Table 2*Deviations From the Partition*

Deviation	Number checked	Profitable	Neutral	Unprofitable	Largest gain
Individual move	581	0	165	416	0
Binary split	5,624	198	0	5,426	0.03175324
Two-coalition merger	28	0	0	28	-0.141959867

Note. The gain of a move is the net payoff change of the moving publisher. The gain of a split or merger is the largest net-payoff change among members of the splitting or merging coalitions, respectively. A deviation is classified as profitable, neutral, or unprofitable when its gain is positive, zero, or negative, respectively. A profitable split or merger need not benefit every participant.

The baseline net payoff in an 11-member coalition is 0.308. After a split into 1 + 10, members of the new 10-member coalition receive 0.3401, and the singleton receives 0.0612. The split does not benefit all participants. The result shows that the payoff maximum cannot guarantee the stability of a nearby integer partition. Identifying a stable partition and incorporating publishers outside the isolated population is left for future work.

6.2 Cost Efficiency with Heterogeneous Publishers

Inside a coalition, joint optimality allocates litigation inversely to the cost coefficient. Holding external spillovers fixed, a given legal-certainty index is therefore weakly cheaper to produce in an all-H coalition than in a mixed or all-L coalition of the same size. This cost

advantage alone does not determine coalition membership or stability. The derivation is provided in Appendix E.

6.3 Discussion and Future Work

This subsection describes two further model extensions that change the baseline structure, and both are left for future work. First, the extension sets the legal-certainty index Θ as dynamic across litigation rounds, which could capture first-mover effect. Second, several competing AI firms may exist instead of a single AI firm monopsonist in the baseline model. The outside option on the publishers' side is therefore a function, changing the two-sided structure.

7. Conclusion

Litigation investment produces legal certainty that raises the AI firm's cost of unauthorized scraping and tightens its outside option. Connecting the public-good channel to alternating-offers bargaining and IS/ES stability shows that coalition size is not independent of the legal environment it helps create. At the baseline scale, the grand coalition is internally and externally stable and robust to coordinated exit. Once the population is large enough for the member-outsider payoff gap to close, the stable size becomes interior and vulnerable to group departure. Several medium-sized coalitions can deliver higher per-member payoffs than a single grand coalition, but do not guarantee stability. The result is consistent with collective-licensing markets: precedent spills over, so the private optimal structure is not necessarily a completely inclusive collective.

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Appendix A

The Bargaining Payoff Rule

A.1 Derivation of $\beta(S)$

If S moves first, S 's equilibrium share is

$$x^*(\Delta t) = \frac{1 - \delta_{AI}(\Delta t)}{1 - \delta_{AI}(\Delta t)\delta_{pub}(\Delta t)} = \frac{1 - e^{-r_{AI}\Delta t}}{1 - e^{-(r_{AI} + r_{pub}(S))\Delta t}}$$

As $\Delta t \rightarrow 0$, the first-mover advantage disappears.

The equilibrium share converges to

$$\beta(S) = \frac{r_{AI}}{r_{pub}(S) + r_{AI}}$$

$\beta(S)$ is increasing in coalition size when $f > 0$.

A.2 Proof of Proposition 1

Let $V = V(\text{data})$, and

$$q = \pi_{\text{scrape}}(\Theta) = V - c^*(\Theta).$$

For a bargaining interval Δt , let $X_{\Delta t}$ and $Y_{\Delta t}$ denote payments to S when S and the AI firm make the offer, respectively. Stationarity and acceptance require

$$Y_{\Delta t} = \delta_{pub}(\Delta t)X_{\Delta t},$$

$$V - X_{\Delta t} = \max\{\delta_{AI}(\Delta t)(V - Y_{\Delta t}), q\}.$$

Substituting for $Y_{\Delta t}$ gives

$$X_{\Delta t} = \min\left\{c^*(\Theta), \frac{1 - \delta_{AI}(\Delta t)}{1 - \delta_{AI}(\Delta t)\delta_{pub}(\Delta t)}V\right\}.$$

Because

$$0 < \delta_{AI}(\Delta t)\delta_{pub}(\Delta t) < 1,$$

this equilibrium payment is unique. As $\Delta t \rightarrow 0$,

$$\frac{1 - \delta_{AI}(\Delta t)}{1 - \delta_{AI}(\Delta t)\delta_{pub}(\Delta t)} \rightarrow \beta(S),$$

and $Y_{\Delta t} \rightarrow X_{\Delta t}$. Hence,

$$\phi_{pub}(\Theta, S) = \min\{c^*(\Theta), \beta(S)V(\text{data})\}.$$

The AI firm receives the remaining surplus,

$$\phi_{AI}(\Theta, S) = V(\text{data}) - \phi_{pub}(\Theta, S).$$

This proves Proposition 1.

A.3 The two-regime classification

Because $c^*(\Theta) \leq k/2$, two regimes arise. If $k \geq 2\beta(S)V(\text{data})$, a finite threshold $\Theta^*(S)$ yields $\phi_{pub} = \beta(S)V(\text{data})$. Otherwise, the outside option always binds. Thus, higher $\beta(S)V(\text{data})$ favors the constrained regime, while higher k favors the recoverable regime.

A.4 The AI firm's avoidance cost

The objective is strictly convex in e , with an unconstrained stationary point

$$e = \frac{D_0\Theta}{k}$$

This is feasible and optimal whenever $\frac{D_0\Theta}{k} \leq 1$. Otherwise, the derivative at $e=1$ is $k -$

$\bar{D}_0\Theta < 0$, so the objective is strictly decreasing on $[0,1]$ and the optimum is the corner solution $e=1$.

Appendix B

Litigation Investment: Spillovers and Optimal Investment

B.1 Proof of Lemma 1

Proof. Individual investment l_i enters $\theta_S^{\text{eff}} = \rho \sum_{j \in N} l_j + (1 - \rho) \sum_{k \in S} l_k$ twice: with weight ρ in the global sum ($i \in N$) and weight $(1 - \rho)$ in internal sum of S ($i \in \text{group}(i)$).

The two weights sum to 1.

For an independent publisher, the first-order characterization is:

$$l_j^* = \min\{l_j^{\text{FOC}}, l_j^{\text{kink}}\}$$

$$\gamma l_j^{\text{FOC}} = c^{*'}(\theta_{\text{eff}\{j\}})$$

with L_j^{kink} the investment level at which $\theta_{\text{eff}\{j\}}$ reaches the saturation threshold $\Theta^*(j)$ holding other publishers' investment fixed.

S 's members jointly choose one common investment level l_S to maximize total payoff:

$$\max_{l_S \geq 0} \phi_S(\theta_S^{\text{eff}}) - \frac{\gamma}{2} \cdot s \cdot l_S^2$$

$$s.t. \theta_S^{\text{eff}} = s l_S + \rho(n - s) l_O$$

Since $\frac{\partial \theta_S^{\text{eff}}}{\partial l_S} = s$, giving the first-order condition:

$$\phi_S'(\theta_S^{\text{eff}}) \cdot s = \gamma \cdot s \cdot l_S$$

$$\phi_S'(\theta_S^{\text{eff}}) = \gamma l_S$$

B.2 Proof of Proposition 3

Proof. Holding other decision-makers' investments fixed, the uncapped net-payoff objective is strictly concave because c^* is concave and litigation costs are strictly convex. Its unique maximizer is zero if the right derivative at zero is nonpositive. Otherwise, it satisfies the stated

first-order condition. If the bargaining cap is reachable, investment beyond the cap cannot increase gross payoff and only increases costs. The optimum is therefore

$$l_g^* = \min\{l_g^{\text{FOC}}, l_g^{\text{kink}}\}, \quad g \in \{O, S\},$$

with the conventions stated in Proposition 3. For the coalition, differentiating gross payoff and total investment costs introduces the same factor s , which cancels.

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Appendix C

Coalition Size Stability

C.1 Proof of Proposition 5

Proof. If $\Delta(2) < 0$, non-increasingness implies that all subsequent entry margins are negative.

Hence $s = 1$ is externally stable, while every size $s \geq 2$ fails internal stability.

Otherwise, let

$$s^* = \max\{s \in \{2, \dots, n\} : \Delta(s) \geq 0\}$$

If $s^* < n$, then $\Delta(s^*) \geq 0$ and $\Delta(s^* + 1) < 0$, so both internal and external stability hold.

If $s^* = n$, internal stability holds because $\Delta(n) \geq 0$, and external stability is vacuous because there are no outsiders. Thus, a stable size exists.

If the sequence is strictly decreasing and has no zero entry, at most one transition from positive to negative entry margins occurs, so the stable size is unique. If $\Delta(j) = 0$, strict decreasingness implies that all preceding entry margins are positive and all subsequent entry margins are negative. Both $j - 1$ and j therefore satisfy the weak stability inequalities, and no other size does.

Table C1*Equilibrium Investment and Payoffs by Coalition Size ($\rho=0.5$)*

s	$\beta(S)$	l_s^*	l_o^*	θ_s^{eff}	θ_o^{eff}	$\pi_{member}(s)$	$\pi_{out}(s)$
1	0.046875	0.078125	0.007717	0.151433	0.116229	0.145117	0.115337
2	0.088235	0.248324	0.000000	0.496647	0.248324	0.191284	0.098329
3	0.125000	0.330962	0.000000	0.992885	0.496443	0.222795	0.089545
4	0.157895	0.408683	0.000000	1.634734	0.817367	0.233310	0.083794
5	0.187500	0.449438	0.000000	2.247191	1.123596	0.224719	0.079588
6	0.214286	0.425532	0.000000	2.553191	1.276596	0.212766	0.076309
7	0.238636	0.404040	0.000000	2.828283	1.414141	0.202020	0.073642
8	0.260870	0.384615	0.000000	3.076923	1.538462	0.192308	0.071407
9	0.281250	0.366972	0.000000	3.302752	1.651376	0.183486	0.069493
10	0.300000	0.350877	0.000000	3.508772	1.754386	0.175439	0.067823
11	0.317308	0.336134	0.000000	3.697479	1.848739	0.168067	0.066348
12	0.333333	0.322581	0.000000	3.870968	1.935484	0.161290	0.065029
13	0.348214	0.310078	0.000000	4.031008	2.015504	0.155039	0.063839
14	0.362069	0.298507	0.000000	4.179104	2.089552	0.149254	0.062756
15	0.375000	0.287770	0.000000	4.316547	2.158273	0.143885	0.061765
16	0.387097	0.277778	0.000000	4.444444	2.222222	0.138889	0.060852
17	0.398438	0.268456	0.000000	4.563758	2.281879	0.134228	0.060006
18	0.409091	0.259740	0.000000	4.675325	2.337662	0.129870	0.059220
19	0.419118	0.251572	0.000000	4.779874	2.389937	0.125786	0.058486
20	0.428571	0.243902	—	4.878049	—	0.121951	—

Note. All parameters are as in Table 1. Payoffs are net of investment costs. All coalition sizes $s=1, \dots, 20$ are reported. Outsider quantities are not applicable at $s=20$.

C.2 Proof of Proposition 6

Proof. For each candidate size $s = 1, \dots, 20$, the litigation-investment equilibrium is solved

under the calibration of Table 1. The resulting net payoffs are

$$\pi_{member}(s) = \frac{1}{s} \min \left\{ c^* \left(\theta_s^{eff}(s) \right), \beta(s) A s^{1/\sigma} \right\} - \frac{\gamma}{2} [l_s^*(s)]^2,$$

and for $s = 1, \dots, 19$,

$$\pi_{\text{out}}(s) = \min \left\{ c^* \left(\Theta_O^{\text{eff}}(s) \right), \beta(1) \frac{A}{\sigma} s^{1/\sigma-1} \right\} - \frac{\gamma}{2} [l_O^*(s)]^2.$$

At $s = 20$, there are no outsiders, so no outsider payoff is required. Table C2 reports every entry margin

$$\Delta(s) = \pi_{\text{member}}(s) - \pi_{\text{out}}(s-1), \quad s = 2, \dots, 20.$$

All entries are strictly positive. The maximum occurs at $s = 4$, and the minimum occurs at $s = 20$:

$$\max_{2 \leq s \leq 20} \Delta(s) = \Delta(4) = 0.143764832,$$

$$\min_{2 \leq s \leq 20} \Delta(s) = \Delta(20) = 0.063465354 > 0.$$

For every $s < 20$,

$$\pi_{\text{member}}(s+1) - \pi_{\text{out}}(s) = \Delta(s+1) > 0.$$

An outsider therefore strictly benefits from joining, so every size below 20 fails external stability.

At $s = 20$, external stability is vacuous. Internal stability holds because

$$\pi_{\text{member}}(20) - \pi_{\text{out}}(19) = \Delta(20) > 0.$$

Hence $s = 20$ is the unique IS/ES-stable size.

Under the stated sequential adjustment rule, an outsider joins at every size below 20. Each entry increases coalition size by one, so the grand coalition is reached after finitely many entries.

Table C2*Sequential Entry Incentives*

$s-1 \rightarrow s$	Coalition Size	$\Delta(s)$	Entry profitable?
1→2	2	0.07594692	Yes
2→3	3	0.12446678	Yes
3→4	4	0.14376483	Yes
4→5	5	0.14092547	Yes
5→6	6	0.13317804	Yes
6→7	7	0.12571142	Yes
7→8	8	0.11866574	Yes
8→9	9	0.11207895	Yes
9→10	10	0.10594607	Yes
10→11	11	0.10024396	Yes
11→12	12	0.09494252	Yes
12→13	13	0.09000991	Yes
13→14	14	0.08541503	Yes
14→15	15	0.08112867	Yes
15→16	16	0.07712392	Yes
16→17	17	0.0733763	Yes
17→18	18	0.06986365	Yes
18→19	19	0.066566	Yes
19→20	20	0.06346535	Yes

Note. Entry margins compare the net payoff from becoming a member of the enlarged coalition with the net payoff from remaining an outsider before entry. Profitability is evaluated using unrounded results.

C.3 Proof of Proposition 7

Proof. The equilibrium net payoff of each member in the grand coalition is

$$\pi_{\text{member}}(20) = 0.121951220.$$

Consider a group of $m \in \{1, \dots, 19\}$ members that leaves and becomes independent publishers. The remaining $20 - m$ members continue as one coalition. After litigation

investments are optimized under the resulting structure, each departing publisher receives

$$\pi_{\text{out}}(20 - m).$$

Define the per-person gain from this deviation as

$$G(m) = \pi_{\text{out}}(20 - m) - \pi_{\text{member}}(20).$$

Table C3 reports the complete deviation check. Every value of $G(m)$ is strictly negative.

The most favorable deviation occurs at $m = 19$, leaving a one-member coalition.

Table C3

Coordinated Exit Payoff

Departing members m	Remaining coalition size $20 - m$	Departing publisher's net payoff $\pi_{\text{out}}(20 - m)$	Gain from exit $G(m)$	Exit profitable?
1	19	0.05848587	-0.063465354	No
2	18	0.05922017	-0.062731051	No
3	17	0.06000648	-0.061944739	No
4	16	0.06085189	-0.061099330	No
5	15	0.06176497	-0.060186250	No
6	14	0.06275623	-0.059194993	No
7	13	0.0638387	-0.058112516	No
8	12	0.06502885	-0.056922368	No
9	11	0.0663478	-0.055603421	No
10	10	0.06782326	-0.054127957	No
11	9	0.06949253	-0.052458694	No
12	8	0.07140729	-0.050543934	No
13	7	0.07364195	-0.048309269	No
14	6	0.07630879	-0.045642434	No
15	5	0.07958792	-0.042363303	No
16	4	0.08379363	-0.038157593	No
17	3	0.08954534	-0.032405877	No
18	2	0.09832851	-0.023622712	No
19	1	0.11533698	-0.006614241	No

Note. Departing publishers become outsiders. Gains use unrounded payoffs.

Appendix D

Larger Population Multi-Coalition Extension

D.1 Derivation of Proposition 8

Proof. $2\beta(S)V_{total}(s) > k$ when s is large enough, so the coalition payment equals $c^*(\theta_S^{\text{eff}})$.

Under $l_0^* = 0$, the interior first-order condition gives

$$\gamma l_S^* = D_0 - \frac{D_0^2}{k} s l_S^*, \quad l_S^* = \frac{k D_0}{\gamma k + D_0^2 s}.$$

Hence

$$\theta_S^{\text{eff}} = \frac{k D_0 s}{\gamma k + D_0^2 s} \rightarrow \frac{k}{D_0}$$

Deducting investment costs gives

$$\begin{aligned} \pi_{\text{member}}(s) &= \frac{c^*(s l_S^*)}{s} - \frac{\gamma}{2} (l_S^*)^2 = \frac{D_0 l_S^*}{s} - \frac{\gamma + D_0^2 s/k}{2} (l_S^*)^2 = \frac{k D_0^2}{2(\gamma k + D_0^2 s)} \\ &= \frac{k}{2s} - \frac{\gamma k^2}{2 D_0^2 s^2} + O(s^{-3}). \end{aligned}$$

For an outsider, $\theta_0^{\text{eff}} = \rho \theta_S^{\text{eff}} \rightarrow \rho k / (D_0)$. Since $\rho > 0$, $c^*(\rho k / D_0) > 0$. In contrast, the payment cap $\beta(1)(A/\sigma)s^{1/\sigma-1}$ tends to zero when $\sigma > 1$. Thus, the outsider payment equals this cap. With $l_0^* = 0$, the payment is also its net payoff. The member payoff decreases faster because $-1 < 1/\sigma - 1 < 0$. Moreover, $\pi_{\text{out}}(s-1) \sim \pi_{\text{out}}(s)$, so $\Delta(s) < 0$ for sufficiently large s . Equating the two leading terms gives the approximation for s_{cross} .

D.2 Numerical Verification of Proposition 9

Proof. Set $n=200$ and retain all other parameters from Table 1. The investment equilibrium is calculated for every $s = 1, \dots, 200$ using the best responses in Appendix B.2. The resulting net

entry margins satisfy

$$\min_{2 \leq s \leq 83} \Delta(s) = \Delta(83) = 0.000018832344754 > 0,$$

$$\max_{84 \leq s \leq 200} \Delta(s) = \Delta(84) = -0.000295927988300 < 0.$$

Every size below 83 fails external stability, and every size above 83 fails internal stability.

Size 83 satisfies both and is IS/ES-stable. Its member net payoff is

$$\pi_{\text{member}}(83) = 0.041753653444676.$$

After a single-member exit,

$$\pi_{\text{out}}(82) = 0.041734821099922 < \pi_{\text{member}}(83).$$

After a two-member exit,

$$\pi_{\text{out}}(81) = 0.041853163210128 > \pi_{\text{member}}(83),$$

giving a per-person gain of 0.000099509765451. All 81 feasible exit cases confirm positive gains

for every $m = 2, \dots, 82$.

D.3 Multi-Coalition Payoffs and Proof of Proposition 10

D.3.1 Proof of Lemma 2

Proof. For a given total investment L_k within coalition S_k , identical quadratic costs imply

$$\sum_{i \in S_k} l_i^2 \geq \frac{L_k^2}{s_k},$$

with equality when all members invest equally. Joint optimality therefore gives a common per-member investment $l_k = L_k/s_k$.

Coalition S_k 's effective legal-certainty index is its own total investment plus the spillover from other coalitions:

$$\Theta_{S_k}^{\text{eff}} = s_k l_k + \rho \sum_{j \neq k} s_j l_j$$

Dividing its bargaining payment by s_k and subtracting each member's investment cost gives the net-payoff formula in Lemma 2.

For equal-sized blocks, symmetry and Shapley efficiency imply

$$\text{Shapley}_k(\pi) = \frac{V_{\text{total}}(n')}{m}$$

At a symmetric investment equilibrium, all members invest l^* , so

$$\Theta^{\text{eff}} = s[1 + \rho(m - 1)]l^*.$$

This proves the lemma.

D.3.2 Symmetric Equilibrium

Write

$$s = \frac{n'}{m}, \quad M(m) = 1 + \rho(m - 1),$$

$$x(m) = \frac{D_0^2}{k} s M(m) = \frac{D_0^2 n'}{k} \left(\rho + \frac{1 - \rho}{m} \right).$$

Interior investment. Holding other coalitions' investments fixed, joint optimality requires

$$c^{*'}(\Theta^{\text{eff}}) = \gamma l.$$

Substituting

$$c^{*'}(\Theta) = D_0 - \frac{D_0^2}{k} \Theta, \quad \Theta^{\text{eff}} = s M(m) l,$$

yields

$$l_{\text{interior}}^*(m) = \frac{D_0}{\gamma + x(m)},$$

$$\Theta_{\text{interior}}^*(m) = \frac{k x(m)}{D_0 [\gamma + x(m)]}.$$

The per-member net payoff is therefore

$$\pi_{\text{interior}}(m) = \frac{c^*(\Theta_{\text{interior}}^*(m))}{s} - \frac{\gamma}{2} [l_{\text{interior}}^*(m)]^2 = \frac{D_0^2 \{M(m)[2\gamma + x(m)] - \gamma\}}{2[\gamma + x(m)]^2}.$$

Capped investment. Define $\Theta_{\text{cap}}(m)$ as the smallest index satisfying

$$c^*(\Theta_{\text{cap}}(m)) = \beta(n'/m) \frac{V_{\text{total}}(n')}{m}.$$

When the right-hand side does not exceed $k/2$,

$$\Theta_{\text{cap}}(m) = \frac{k}{D_0} \left[1 - \sqrt{1 - \frac{2\beta(n'/m)V_{\text{total}}(n')}{km}} \right].$$

If the payment cap exceeds $k/2$, set $\Theta_{\text{cap}}(m) = +\infty$

On the capped branch,

$$l_{\text{cap}}^*(m) = \frac{\Theta_{\text{cap}}(m)}{sM(m)}$$

$$\pi_{\text{cap}}(m) = \beta(n'/m) \frac{V_{\text{total}}(n')}{n'} - \frac{\gamma}{2} [l_{\text{cap}}^*(m)]^2.$$

The equilibrium net payoff is

$$\pi_{\text{member}}(m) = \begin{cases} \pi_{\text{interior}}(m), & \Theta_{\text{interior}}^*(m) \leq \Theta_{\text{cap}}(m) \\ \pi_{\text{cap}}(m), & \Theta_{\text{interior}}^*(m) > \Theta_{\text{cap}}(m) \end{cases}$$

At equality, the investment levels and net payoffs of the two branches coincide.

D.3.3 Proof of Proposition 10

Proof. Throughout the continuous relaxation,

$$M'(m) = \rho \geq 0, \quad x'(m) = -\frac{D_0^2 n'(1-\rho)}{km^2} < 0.$$

Part (i). Differentiating the interior net payoff gives

$$\pi'_{\text{interior}}(m) = \frac{D_0^2 [2\gamma + x(m)]}{2[\gamma + x(m)]^2} M'(m) + \frac{D_0^2 \{2\gamma - M(m)[3\gamma + x(m)]\}}{2[\gamma + x(m)]^3} x'(m).$$

The first term is nonnegative. Since $M(m) \geq 1$, the coefficient of $x'(m)$ is negative, making

the second term strictly positive. Hence

$$\pi'_{\text{interior}}(m) > 0.$$

Part (ii). The interior gross payoff per member is

$$\frac{c^*(\theta_{\text{interior}}^*(m))}{s} = D_0^2 M(m) g(x(m)), \quad g(x) = \frac{2\gamma + x}{2(\gamma + x)^2}.$$

Because

$$g'(x) = -\frac{3\gamma + x}{2(\gamma + x)^3} < 0,$$

this gross payoff strictly increases with m . In contrast, the payment cap per member,

$$\beta(n'/m) \frac{V_{\text{total}}(n')}{n'},$$

strictly decreases with m .

These two quantities can therefore cross at most once. The interior branch applies below the crossing and the capped branch above it, proving part (ii).

Part (iii). At interior transition m_c ,

$$\theta_{\text{interior}}^*(m_c) = \theta_{\text{cap}}(m_c).$$

The two branches therefore have the same investment and net payoff at m_c . By part (i), the active interior net payoff increases up to m_c . Under the additional condition in part (iii), the active capped net payoff strictly decreases thereafter. Thus, m_c is the unique maximum.

The additional condition concerns the derivative

$$\pi'_{\text{cap}}(m) = -\frac{r_{AI} f V_{\text{total}}(n')}{n'^2 [r_0 + r_{AI} + f m / n']^2} - \gamma l_{\text{cap}}^*(m) \frac{dl_{\text{cap}}^*(m)}{dm}.$$

If investment decreases with m , the second term is positive. Therefore, decreasing gross payments alone do not establish decreasing net payoffs.

D.3.4 Numerical Illustration

Under the calibration of Table 1 with $n' = 83$, the equal-size formulas evaluated at integer coalition counts $m = 1, \dots, 83$ yield increasing net payoffs through $m = 8$ and decreasing net payoffs thereafter. The maximum on this grid is

$$\pi_{\text{member}}(8) = 0.341975009$$

Table D1 reports selected values. These calculations allow fractional block sizes and do not establish the stability of an integer partition.

Table D1

Per-member net payoffs under the equal-size approximation

m	$s = 83/m$	Θ^{eff}	Active branch	π_{member}
1	83	6.931106	Interior	0.04175365
2	41.5	6.635576	Interior	0.08449175
4	20.75	6.416699	Interior	0.17297991
6	13.833333	6.327253	Interior	0.26285217
7	11.857143	6.29981	Interior	0.30799808
8	10.375	5.709578	Capped	0.34197501
9	9.222222	4.079826	Capped	0.32332323
10	8.3	3.19909	Capped	0.30443914
20	4.15	0.820617	Capped	0.18731349
40	2.075	0.221346	Capped	0.10517561
83	1	0.054288	Capped	0.05410238

Notes All parameters other than population size are as in Table 1. Payoffs are net of litigation-investment costs. Selected rows are shown from the complete calculation over $m = 1, \dots, 83$.

The equal-size approximation permits fractional coalition sizes.

D.4 Proof of Proposition 11

D.4.1 Enumeration and Equilibrium Calculation

The partition contains three 11-member coalitions and five 10-member coalitions. Each publisher can move to any of the other seven coalitions, giving

$$83 \times 7 = 581$$

individual moves.

A coalition with s labelled members has $2^{s-1} - 1$ unordered divisions into two nonempty groups. Checking every such division in every source coalition gives

$$3(2^{10} - 1) + 5(2^9 - 1) = 5624$$

binary splits. All pairs of coalitions are checked for merger.

For each resulting partition π' , Shapley values and all coalitions' equilibrium investments are calculated. Net payoffs are evaluated using Lemma 2, including the investment costs of the participating publishers. Coalitions whose memberships have not changed are also included in the equilibrium calculation.

The parameters are

$$r_{AI} = 0.15, \quad r_0 = 0.05, \quad f = 3, \quad D_0 = 1, \quad k = 8,$$

$$A = 3.2, \quad \sigma = 1.3, \quad \gamma = 1.6, \quad \rho = 0.5.$$

The 83 publishers are considered in isolation, without investment spillovers from publishers outside this population.

Homogeneity makes payoffs depend on resulting coalition sizes rather than member identities. For a specified source coalition, a split into $a < b$ has $\binom{s}{a}$ labelled realizations; when $a = b$, the count is $\binom{s}{a}/2$. Table D2 groups equivalent cases and reports their full multiplicities.

D.4.2 Complete Payoff Comparisons and Proof

The baseline net payoffs are $\pi_{\text{member},11}(\pi) = 0.308364774$, $\pi_{\text{member},10}(\pi) = 0.342872308$.

Table D2

All Payoff-distinct Deviations from the 83-publisher Partition

Change	Number represented	Net payoff after change	Largest gain
Move 11 → 11	66	0.28102493	-0.027339840
Move 11 → 10	165	0.30836477	0
Move 10 → 11	150	0.28179346	-0.061078847
Move 10 → 10	200	0.31011761	-0.032754695
Split 11 → 1 + 10	33	0.061160412 / 0.340118014	0.03175324
Split 11 → 2 + 9	165	0.110383317 / 0.325851995	0.01748722
Split 11 → 3 + 8	495	0.152866293 / 0.303764052	-0.004600721
Split 11 → 4 + 7	990	0.190161238 / 0.279559694	-0.028805080
Split 11 → 5 + 6	1,386	0.223256747 / 0.252871587	-0.055493186
Split 10 → 1 + 9	50	0.061164694 / 0.326235305	-0.016637002
Split 10 → 2 + 8	225	0.110399611 / 0.304028762	-0.038843546
Split 10 → 3 + 7	600	0.152901563 / 0.279750023	-0.063122284
Split 10 → 4 + 6	1,050	0.190222291 / 0.253007718	-0.089864590
Split 10 → 5 + 5	630	0.223350914 / 0.223350914	-0.119521394
Merge 11 + 11	3	0.1580115	-0.150353270
Merge 10 + 11	15	0.16640491	-0.141959867
Merge 10 + 10	10	0.17598688	-0.166885432

Note. In a move $a \rightarrow b$, b is the destination's initial size and payoff is the mover's; merger gains use the lower initial payoff.

Proof of Proposition 11. Every individual move has a nonpositive gain. The 165 neutral moves transfer a publisher from an 11-member coalition to a 10-member coalition, exchanging the two sizes and leaving the mover's net payoff unchanged. All other moves are strictly unprofitable. Every merger reduces the net payoff of every participating member.

Exactly two binary-split patterns yield a positive gain: $11 \rightarrow 1 + 10$ and $11 \rightarrow 2 + 9$. The larger resulting groups receive 0.340118014 and 0.325851995, respectively, exceeding their members' original payoff of 0.308364774. These patterns represent

$$3 \binom{11}{1} + 3 \binom{11}{2} = 198$$

labelled splits. They violate Definition 3(ii), so the partition is not merge/split stable. The smaller groups lose from both splits. The result does not assert unanimous agreement to split.

D.4.3 Additional Multiway-Split Check

Beyond the binary splits in Definition 3, every integer partition of each source-coalition size into three or more positive parts is checked. For group sizes $a_1, \dots, a_q$ summing to s , the number of labelled divisions represented by a size profile is

$$\frac{s!}{\prod_{j=1}^q a_j! \prod_t r_t!},$$

where r_t counts groups of size t .

The 86 source-size/split-size profiles represent 2,609,953 labelled multiway splits. Only $11 \rightarrow 1 + 1 + 9$ produces a positive gain, representing 165 labelled divisions. Members of the 9-member group gain 0.017153819 each, while the two singletons lose. Allowing these additional splits therefore leaves the instability conclusion unchanged.

D.4.4 Numerical Verification and Scope

The 103 distinct partitions arising from the baseline and all checks are independently solved using two methods: double-precision Shapley subset counting with a scalar aggregate-investment root, and 60-digit decimal Shapley enumeration with successive coalition best responses. The maximum difference in per-member net payoffs is 2.78×10^{-16} , and the largest decimal best-response residual is below 7.41×10^{-43} .

Shapley efficiency, symmetry, and enumeration counts are verified. Gains are classified using high-precision results with tolerance 10^{-25} . Neutral moves also follow from symmetry. These checks apply to the specified partition and deviation rules.

Appendix E

Heterogeneous Publishers: Robustness Check

Consider a coalition S with n_H^S type-H and n_L^S type-L members. Their litigation-cost coefficients satisfy $0 < \gamma_H < \gamma_L$, so type-H members have lower litigation costs for the same investment level. Members jointly choose type-specific investment levels l_H and l_L to maximize the net payoff

$$\varphi_S(\Theta_S^{\text{eff}}) - (\gamma_H/2)n_H^S l_H^2 - (\gamma_L/2)n_L^S l_L^2$$

subject to

$$\Theta_S^{\text{eff}} = n_H^S l_H + n_L^S l_L + \rho \sum_{j \notin S} l_j, \quad l_H, l_L \geq 0,$$

holding outsiders' investments fixed.

When both types are present, a positive interior optimum in the differentiable, uncapped region satisfies

$$\gamma_H \cdot l_H^* = \gamma_L \cdot l_L^* = \varphi'_S(\Theta_S^{\text{eff}})$$

Or equivalently, $\frac{l_H^*}{l_L^*} = \frac{\gamma_L}{\gamma_H} > 1$.

Here, l_H^* and l_L^* denote the jointly optimal per-member investment levels of the two types.

The ratio exceeds one because $\gamma_H < \gamma_L$, which means type-H members invest more at a positive interior optimum.

Investment is allocated inversely to the type-specific cost coefficient. The symmetric first-order condition in Proposition 3 emerges as the special case where $\gamma_H = \gamma_L$.