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Paper Title: Capital Misallocation and Shock Amplification:
A Quantitative Model of Ownership-Based
Financing Disparities in China

Capital Misallocation and Shock Amplification: A Quantitative Model of Ownership-Based Financing Disparities in China

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Abstract

This paper investigates how asymmetric financing constraints between SOEs and private firms in China affect capital allocation and aggregate output, and whether tighter financing constraints amplify the effects of a negative productivity shock. I build a static equilibrium model involving private firms and SOEs and compare the decentralized equilibrium under financing constraints with the efficient allocation without firm-level financing constraints. The baseline results show that capital misallocation caused by asymmetric financing constraints reduces aggregate output by 4.86% relative to the pre-shock efficient output. A 10% negative productivity shock produces a direct output loss of 9.51% of pre-shock efficient output. A further decrease in private firms' financing capacity generates an additional amplification loss of 0.62%, bringing the total output loss to 14.99%. These results show that asymmetric financing constraints cause capital misallocation and amplify output losses when negative shocks further restrict private firms' access to capital.

Keywords: SOE-private firm financing disparity, financing constraints, capital misallocation, financial amplification

JEL: D24, D50, E44, G32

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1 Introduction

Asymmetric financing constraints can affect both firms' access to capital and the allocation of capital across firms. When firms with higher productivity face tighter financing constraints, capital may not flow to the firms with the highest marginal product of capital, reducing aggregate output even if the total capital in the economy remains unchanged. This allocation problem may become more serious during economic contractions because a negative shock can further weaken firms' financing capacity and amplify the initial output loss.

In China, state-owned enterprises (SOEs) generally obtain external financing more easily because they have government backing and closer relationships with state-owned banks. In contrast, private firms often face tighter financing constraints and have less access to capital despite potentially higher productivity. This problem motivates my first research question: how do asymmetric financing constraints between private firms and SOEs affect capital allocation, and how much aggregate output is lost as a result?

The paper also investigates the response to a negative productivity shock. If the shock reduces private firms' financing capacity, their financing constraints become tighter, and the amount of capital they can obtain will further decrease. Since private firms are assumed to have higher productivity in this model, this further misallocation creates additional output losses. My second research question is therefore: does the tightening of private firms' financing constraints amplify the output effect of a negative productivity shock, and if so, by how much?

The first mechanism operates through capital misallocation: tighter financing constraints limit capital access for more productive private firms, causing more capital to remain with SOEs even when the marginal product of capital of an SOE is lower than that of a private firm, thereby reducing aggregate output. Cong et al. (2019) find that

SOEs received larger credit increases than private firms during China's economic stimulus, consistent with SOEs' financing advantage. The second mechanism resembles the financial accelerator described by Bernanke et al. (1999): negative shocks weaken firms' balance sheets and access to financing, further reducing investment and output. In this model, tighter financing constraints for private firms also worsen capital misallocation, amplifying the initial output loss.

This paper establishes a static equilibrium model in which private firms and SOEs produce using capital and labor. Private firms are more productive but face tighter financing constraints, while SOEs are less productive but can obtain more external financing. I compare the decentralized equilibrium under financing constraints with a social planner's efficient allocation, which allocates the same total supplies of capital and labor according to productivity and marginal products without being affected by firm-level financing constraints.

I measure capital misallocation using the marginal product of capital gap between firm types and output loss relative to efficient output. I then introduce a negative productivity shock. The analysis first holds the financing capacity unchanged to isolate the direct effect, then reduces private firms' financing capacity to measure the amplification effect. Thus, the post-shock output loss is divided into three parts: pre-existing misallocation loss, the direct output loss from a negative productivity shock, and the amplification loss from tighter private-firm financing constraints.

The baseline results show that private firms' financing constraints lead to capital misallocation, reducing pre-shock output by 4.86% relative to efficient output. A 10% negative productivity shock causes a direct output loss of 9.51% of the pre-shock efficient output, while reduced private firms' financing capacity adds an additional amplification loss of 0.62%. Together, these effects bring the total output loss relative to pre-shock efficient output to 14.99%. Comparative statics show that larger financing disparities increase the

MPK gap and output loss, with the main result remaining robust across the parameter ranges considered.

This paper makes three main contributions. First, it models how financing capacity disparities between private firms and SOEs in China affect capital allocation. Second, it quantifies capital misallocation and aggregate output loss using the MPK gap and the difference between decentralized and efficient output. Third, it divides the post-shock output loss into the pre-shock misallocation loss, the direct productivity loss, and the financial amplification loss.

The remainder of the paper proceeds as follows. Section 2 reviews empirical evidence on financial disparities in China. Section 3 presents the model, decentralized equilibrium, and efficient allocation. Section 4 analyzes the theoretical results of capital misallocation, aggregate output loss, and the financial amplification. Section 5 quantifies these mechanisms through a baseline calibration and comparative statics. Section 6 concludes the paper.

2 Empirical Motivation

This section reviews empirical evidence supporting the model’s baseline assumptions and the financing response underlying the amplification mechanism.

2.1 Financing Constraints of Private Firms and SOEs

Existing research has found that Chinese banks treat private firms differently during credit allocation. [Brandt and Li \(2003\)](#) use matched Chinese bank–firm data to investigate banks’ lending decisions toward private firms. They find that banks treat private firms differently in ways not fully explained by firms’ risks or information asymmetry. This evidence suggests that firm ownership affects access to financing in a financial system dominated by state-owned banks.

Other research further illustrates that political connections and government support also contribute to this disparity. [Lu et al. \(2012\)](#) find that, compared with SOEs, non-SOEs are more likely to hold shares in commercial banks, which may help them overcome discrimination they face in the formal credit market. Similarly, [Cull et al. \(2015\)](#) use a representative sample of Chinese firms and find that government connections can significantly affect the financing conditions faced by firms. Firms with stronger government connections normally face weaker financing constraints. Studies focusing on private firms also reach similar conclusions. [Cheng and Wu \(2019\)](#) use data from the 2012 Nationwide Survey of Private Enterprises in China and find that politically connected private firms are more likely to obtain commercial bank loans, particularly among small firms and those operating in industries with a larger SOE presence.

More direct evidence comes from China's economic stimulus program from 2009 to 2010. Matching loan-level data from nineteen large banks with manufacturing-firm data, [Cong et al. \(2019\)](#) find that credit expansion disproportionately benefited SOEs. A one-standard-deviation increase in credit supply raised borrowing by 15.7% for fully state-owned firms but only 11.3% for fully private firms, making the SOE response approximately 39% larger. This difference persists after controlling for initial productivity, suggesting that credit allocation was not driven solely by productivity.

[Cong et al. \(2019\)](#) also find that approximately 70% of the resulting increase in capital misallocation came from capital flowing from private firms to SOEs, while the remaining 30% flowed toward low-productivity private firms. This supports both the SOE financing advantage and the paper's mechanism: shifting credit from constrained, more productive private firms toward less productive SOEs widens the MPK gap and reduces aggregate output.

Motivated by this evidence, the model imposes the following assumption.

Assumption 1. Let λ_g denote the financing multiple of type- g firms, measuring their

maximum financeable capital per unit of net worth. Then

$$\lambda_S > \lambda_P,$$

where S denotes SOEs and P denotes private firms.

This restriction represents an average financing advantage conditional on net worth. It does not imply that every SOE borrows or uses more capital than every private firm. Section 3 formalizes this restriction through the firm-level financing constraint.

2.2 Financial Conditions after a Negative Shock

This subsection examines whether negative economic shocks tighten firms' financing conditions and whether private firms are more affected than SOEs. The financial-accelerator literature explains this mechanism through firms' balance sheets. [Bernanke et al. \(1999\)](#) point out that when economic shocks lower firms' expected revenues, profits, and net worth, collateral values also decline. Because of information asymmetry between firms and external investors, lower net worth increases the cost of external financing and reduces firms' ability to obtain finance. Therefore, firms that initially face tighter financing constraints should respond more strongly to negative shocks.

Empirical evidence supports this prediction. [Gertler and Gilchrist \(1994\)](#) examine American manufacturing firms following monetary policy contractions and find that the sales, inventories, and short-term debt of small firms decline more than those of large firms. During the 2008–2009 global financial crisis, financially constrained firms were also more likely to reduce investment, technological expenditure, and employment ([Campello et al., 2010](#)). Similarly, [Duchin et al. \(2010\)](#) find that the external credit-supply shock significantly reduced investment, particularly among firms with low cash reserves, high net debt, and greater reliance on external financing. These findings show that firms with

weaker initial financing capacity are more sensitive to contractionary shocks.

Chinese evidence further indicates that this sensitivity differs by ownership. Using Chinese firm-level data from 1998 to 2007, [Shi et al. \(2019\)](#) find that when market liquidity tightens, private firms face more severe credit constraints than SOEs, placing greater restrictions on their growth. When liquidity becomes more abundant, private firms face fewer financing constraints and grow faster than SOEs.

The sudden economic contraction caused by COVID-19 provides additional evidence of this disparity. Using transaction-level data on Chinese SMEs' applications for lines of credit, [Liu et al. \(2022\)](#) compare firms in Hubei with those in other provinces. They find that the probability and frequency of credit applications and the total amount of credit requested decreased significantly after the outbreak. This effect was more pronounced among non-SOEs and firms without prior relationships with banks. State-owned banks also reduced collateral requirements and increased credit approval rates, partially alleviating but not eliminating the financing shock.

Using data on Chinese publicly listed companies, [Liao et al. \(2023\)](#) find that COVID-19 increased firms' KZ financing constraint index and the cost of debt financing while reducing commercial credit. These findings show that negative shocks not only reduce firms' current income but also weaken their ability to obtain external financing. They are therefore consistent with the model's assumption that a negative productivity shock is accompanied by a decline in financing capacity.

Using survey data from managers of 1,182 Chinese firms, [Wu and Xu \(2021\)](#) find that SOEs experienced smaller reductions in business activities than non-SOEs during COVID-19. However, this relative advantage disappeared after controlling for government support. This suggests that SOEs' greater stability during negative shocks reflects government support rather than higher productivity.

In addition, [Wang and Yu \(2023\)](#) find that SOEs provided more trade credit than

domestic private firms after the outbreak, indicating stronger liquidity and credit-supply capacity. [Zeng and Tang \(2023\)](#) also find that private firms with supply-chain relationships with SOEs obtained more bank loans. These findings show that relationships with SOEs can alleviate financing difficulties during large negative shocks, while private firms without government or SOE connections remain more vulnerable.

Taken together, these studies suggest that negative shocks reduce firms' revenues, profits, net worth, and collateral values, thereby tightening external financing conditions. Chinese private firms are more vulnerable because they face tighter initial constraints, while SOEs maintain relatively stable financing through government support, implicit guarantees, and relationships with state-owned banks. This motivates the following assumption.

Assumption 2 (Baseline financial response). *Let $t = 0$ and $t = 1$ denote the pre-shock and post-shock states, respectively. Let $\varepsilon > 0$ denote the size of the negative productivity shock, λ_{gt} the financing multiple of type- g firms in state t , and $\phi_P > 0$ the sensitivity of private firms' financing capacity to the shock. Assuming $0 < \phi_P \varepsilon < 1$ and $(1 - \phi_P \varepsilon) \lambda_{P0} \geq 1$,*

$$\lambda_{P1} = (1 - \phi_P \varepsilon) \lambda_{P0} < \lambda_{P0}, \quad \lambda_{S1} = \lambda_{S0}.$$

Assumption 2 is a reduced-form parameterization of the average disparity documented in existing literature. The evidence motivates a larger decline in private firms' financing capacity than in that of SOEs. The proportional response and $\lambda_{S1} = \lambda_{S0}$ are simplifying assumptions. In particular, the latter normalizes the SOE financing response to zero in the baseline model. It does not claim that SOE financing conditions remain unchanged in reality. In the model, this tightening in financing conditions is captured through changes in λ_P rather than changes in firm net worth.

3 Model

This section builds a static model to examine how asymmetric financing constraints affect capital allocation between private firms and SOEs. It first solves the decentralized equilibrium under financing constraints and then compares it with the efficient allocation based on the same total supplies of capital and labor.

3.1 Model Setting

The economy consists of a unit mass of firms and a unit mass of workers. Firms belong to one of two types, denoted by $g \in \{P, S\}$, where P denotes private firms and S denotes SOEs. Private firms account for a share s of the total mass of firms, while SOEs account for the remaining share $1 - s$. Firms within each type are identical, and k_g and l_g denote the capital and labor inputs of a representative type- g firm.

3.1.1 Aggregate resources

Because this is a static model, this paper does not directly model household saving or capital accumulation. Instead, I assume that the economy has a fixed aggregate capital supply $\bar{K}$. A financial sector channels the available capital to private firms and SOEs, but I do not explicitly model its optimization problem in order to keep the model simple and focus on the results that this paper is interested in.

All firms face the same capital user cost r . This cost is endogenous and is determined jointly by firms' optimal choices and the capital-market-clearing condition.

The economy also contains a unit mass of identical workers. Each worker supplies one unit of labor inelastically, so aggregate labor supply is normalized to one.

Since household decisions are not central to the mechanism studied in this paper, the household's optimization problem is not explicitly modeled. The wage is determined by

firms' optimal labor demand and the labor-market-clearing condition.

Therefore, the aggregate resource constraints are

$$sk_P + (1 - s)k_S = \bar{K}, \quad sl_P + (1 - s)l_S = \bar{L} = 1. \quad (1)$$

These conditions require the total capital and labor used by private firms and SOEs to equal the aggregate supplies of capital and labor.

3.2 Firms

3.2.1 Production Technology

Both types of firms use capital and labor and share the same production function:

$$y_g = Az_g k_g^\alpha l_g^\beta, \quad g \in \{P, S\}.$$

Here, y_g is the output of a type- g firm. A represents aggregate productivity, while z_g represents type-specific productivity. Moreover, k_g and l_g represent the capital and labor used by the firm, and α and β are their respective elasticities.

I assume that private firms are more productive than SOEs:

$$z_P > z_S.$$

Holding aggregate productivity and input use fixed, private firms therefore produce more output and have higher marginal products of capital and labor than SOEs. This productivity advantage creates an efficient motive for allocating more capital and labor to private firms.

The production technology exhibits decreasing returns to scale:

$$\alpha > 0, \quad \beta > 0, \quad \alpha + \beta < 1.$$

Thus, output increases less than proportionally when capital and labor expand by the same proportion.

Decreasing returns to scale capture fixed factors that are not explicitly modeled and ensure that each firm has a finite optimal scale. They also imply declining marginal products as a firm expands. Therefore, although private firms are more productive, it remains efficient for both firm types to operate (Restuccia and Rogerson (2008); Moll (2014)).

3.2.2 Financial Constraints

Although aggregate capital supply is fixed, firms differ in how much capital they can obtain and use. A type- g firm faces the financing constraint

$$k_g \leq \lambda_g a_g, \quad g \in \{P, S\}.$$

Here, a_g represents the net worth of a type- g firm. This net worth can be used as internal funds or as collateral for obtaining external financing. The parameter $\lambda_g \geq 1$ represents the financing multiple. Thus, $\lambda_g a_g$ represents the maximum total amount of capital that the firm can finance and use.

When $k_g > a_g$, the firm's capital use exceeds its net worth. The firm must then borrow externally to finance the difference. Thus, the firm's external financing can be written as

$$b_g = \max\{0, k_g - a_g\}.$$

If the financing constraint binds, so that $k_g = \lambda_g a_g$, the maximum amount of external

financing is

$$b_g^{\max} = \lambda_g a_g - a_g = (\lambda_g - 1)a_g.$$

It is important to distinguish between the aggregate capital supply $\bar{K}$ and the individual firm's financing limit $\lambda_g a_g$. The aggregate capital supply $\bar{K}$ represents the total amount of capital available in the economy, whereas $\lambda_g a_g$ restricts the amount of capital that an individual type- g firm can finance and use. To capture the financing advantage of SOEs, I impose Assumption 1.

SOEs have stronger government support, which makes it easier for them to obtain external financing. This inequality does not mean that every SOE uses more capital than every private firm in equilibrium. Instead, it means that, holding all other conditions fixed, particularly firms' net worth, an SOE can finance and use more capital than a private firm.

3.2.3 Profit Maximization

Taking the wage w and the user cost of capital r as given, each type- g firm chooses its capital and labor inputs, k_g and l_g , to maximize profit subject to financial constraints. The firm's problem is

$$\begin{aligned} \max_{k_g, l_g} \quad & Az_g k_g^\alpha l_g^\beta - w l_g - r k_g \\ \text{subject to} \quad & k_g \leq \lambda_g a_g, \\ & k_g, l_g \geq 0, \quad g \in \{P, S\}. \end{aligned}$$

Here, the price of the final good is normalized to one. Thus, $Az_g k_g^\alpha l_g^\beta$ represents both the firm's output and its total revenue. The term $w l_g$ represents the total labor cost, where w is the wage faced by all firms. The term $r k_g$ represents the total cost of capital, where r is the user cost of capital.

Capital k_g and labor l_g are the firm's choice variables. The variables A , z_g , w , r , a_g , and λ_g are taken as given by each individual firm.

Even if part of the capital comes from the firm's internal funds and another part comes from external financing, all capital faces the same user cost r . If internally and externally financed capital had different costs, firms could borrow capital at a lower cost and invest their own capital at a higher rate of return, thereby earning an arbitrage profit. Thus, under the no-arbitrage condition, every unit of capital faces the same user cost r , regardless of whether it is financed internally or externally.

3.2.4 Optimality Conditions

The firm's optimal capital and labor choices are characterized by the following proposition. The complete Lagrangian and proof are provided in the Appendix.

Proposition 1 (Firm Optimality under Financing Constraints). *For each firm type $g \in \{P, S\}$, the firm's optimal choices are characterized by:*

$$\begin{aligned} MPL_g &= \beta A z_g k_g^\alpha l_g^{\beta-1} = w, \\ MPK_g &= \alpha A z_g k_g^{\alpha-1} l_g^\beta = r + \mu_g, \\ \mu_g &\geq 0, \\ \mu_g(\lambda_g a_g - k_g) &= 0. \end{aligned}$$

The first condition states that the firm equates the marginal product of labor with the wage. The second shows that the financing constraint creates a wedge, denoted by μ_g , between the marginal product and the user cost of capital.

μ_g is the shadow value of the firm's financing constraint. It measures how much additional profit the firm can generate when its financing limit is relaxed by one additional unit. Thus, μ_g can also be understood as the capital wedge created by the financing constraint

in the firm's capital choice. A higher μ_g means that the financing constraint has a stronger effect on the firm's capital decision. Complementary slackness implies that $\mu_g = 0$ when the constraint is slack. Therefore, $MPK_g = r$. When the constraint binds and restricts the firm's capital choice, $\mu_g > 0$, so $MPK_g > r$.

3.3 Decentralized Market Equilibrium

A decentralized market equilibrium consists of capital and labor allocations,

$$\{k_P^D, k_S^D, l_P^D, l_S^D\},$$

and prices,

$$\{w^D, r^D\},$$

such that: (i) given prices, private firms and SOEs maximize their profits subject to their financing constraints; and (ii) the capital and labor markets clear.

3.3.1 Capital Allocation

As a baseline, I assume that the financing constraint of private firms is binding, while the financing constraint of SOEs is non-binding. Thus,

$$k_P^D = \lambda_P a_P, \quad \mu_P^D > 0, \quad (2)$$

while

$$k_S^D < \lambda_S a_S, \quad \mu_S^D = 0.$$

Combining the binding private-firm constraint in Equation (2) with capital-market clearing in Equation (1) gives the equilibrium capital allocation

$$k_P^D = \lambda_P a_P, \quad k_S^D = \frac{\bar{K} - s\lambda_P a_P}{1 - s}.$$

To ensure that the total capital supply can be allocated without violating firms' financing constraints, I impose the following feasibility condition:

$$s\lambda_P a_P < \bar{K} < s\lambda_P a_P + (1 - s)\lambda_S a_S.$$

The lower bound ensures that SOEs use a positive amount of capital, while the upper bound ensures that their financing constraint is non-binding. The additional condition

$$MPK_P^D > MPK_S^D$$

confirms that the private-firm financing constraint is binding in equilibrium. The baseline calibration confirms that this condition is satisfied.

3.3.2 Labor Allocation

The labor-market-clearing condition is given by the second condition in Equation (1). Because both firm types face the same equilibrium wage and their financing constraints do not directly restrict labor, their labor choices satisfy

$$MPL_P^D = MPL_S^D = w^D.$$

Substituting the marginal products of labor, I obtain

$$\beta A z_P (k_P^D)^\alpha (l_P^D)^{\beta-1} = \beta A z_S (k_S^D)^\alpha (l_S^D)^{\beta-1}.$$

Define

$$q_P^D = [z_P(k_P^D)^\alpha]^\frac{1}{1-\beta}, \quad q_S^D = [z_S(k_S^D)^\alpha]^\frac{1}{1-\beta}. \quad (3)$$

Combining this relationship with the labor-market-clearing condition, I obtain

$$l_P^D = \frac{q_P^D}{sq_P^D + (1-s)q_S^D}, \quad l_S^D = \frac{q_S^D}{sq_P^D + (1-s)q_S^D}.$$

The equilibrium wage is

$$w^D = \beta A z_P (k_P^D)^\alpha (l_P^D)^{\beta-1} = \beta A z_S (k_S^D)^\alpha (l_S^D)^{\beta-1}.$$

Since the financing constraint of SOEs is non-binding, $\mu_S^D = 0$. Therefore, the capital optimality condition of SOEs determines the equilibrium user cost of capital:

$$r^D = MPK_S^D = \alpha A z_S (k_S^D)^{\alpha-1} (l_S^D)^\beta.$$

The capital wedge faced by private firms is therefore

$$\mu_P^D = MPK_P^D - r^D > 0.$$

3.3.3 Aggregate Output

Aggregate output in the decentralized market equilibrium is

$$Y^D = sy_P^D + (1-s)y_S^D.$$

This means that aggregate output is the weighted sum of the output produced by private

firms and SOEs. Substituting the production functions, I obtain

$$Y^D = A [sz_P(k_P^D)^\alpha(l_P^D)^\beta + (1-s)z_S(k_S^D)^\alpha(l_S^D)^\beta].$$

For simplicity, using the definitions of q_P^D and q_S^D , aggregate output can be simplified as

$$Y^D = A [sq_P^D + (1-s)q_S^D]^{1-\beta}. \quad (4)$$

3.4 Efficient allocation

To measure the output loss caused by financing constraints, I construct an efficient allocation as a benchmark. A social planner allocates the same aggregate supplies of capital and labor as in the decentralized equilibrium but is not subject to firm-specific financing constraints. Therefore, this benchmark allows us to examine how aggregate output changes when only the allocation of resources changes, while the total amount of resources remains the same.

The social planner solves:

$$\begin{aligned} \max_{\{k_P^E, k_S^E, l_P^E, l_S^E\}} \quad & Y^E = A [sz_P(k_P^E)^\alpha(l_P^E)^\beta + (1-s)z_S(k_S^E)^\alpha(l_S^E)^\beta], \\ & sk_P^E + (1-s)k_S^E = \bar{K}, \\ & sl_P^E + (1-s)l_S^E = 1, \\ & k_P^E, k_S^E, l_P^E, l_S^E \geq 0. \end{aligned}$$

Unlike the decentralized equilibrium, the planner's problem does not include the firm-specific financing constraints $k_g^E \leq \lambda_g a_g$.

3.4.1 Conditions for Efficient Allocation

Proposition 2 (Efficient Allocation). *At an interior solution, the social planner equalizes the marginal products of capital and labor across the two firm types:*

$$MPK_P^E = MPK_S^E, \quad MPL_P^E = MPL_S^E.$$

Under the efficient allocation, the two types of firms have the same marginal product of capital and the same marginal product of labor. If there is still a difference in the marginal product of a resource between the two types of firms, the social planner will transfer this resource from the firm with the lower marginal product to the firm with the higher marginal product, thereby increasing aggregate output. Thus, as long as there is a disparity in marginal products, the allocation is not efficient. Because the production function has decreasing returns to scale, even if private firms have higher productivity, the social planner will not allocate all resources to private firms. The social planner will continue to reallocate resources until the two types of firms have the same MPK and the same MPL.

From the efficient optimality conditions, I can derive the relationship between the capital and labor allocations of the two types of firms. The equality of the marginal products of capital and labor implies that

$$\begin{aligned} z_P(k_P^E)^{\alpha-1}(l_P^E)^\beta &= z_S(k_S^E)^{\alpha-1}(l_S^E)^\beta, \\ z_P(k_P^E)^\alpha(l_P^E)^{\beta-1} &= z_S(k_S^E)^\alpha(l_S^E)^{\beta-1}. \end{aligned}$$

Combining these conditions gives

$$\rho \equiv \frac{k_P^E}{k_S^E} = \frac{l_P^E}{l_S^E} = \left(\frac{z_P}{z_S} \right)^{\frac{1}{1-\alpha-\beta}}.$$

Since $z_P > z_S$, I have $\rho > 1$. This means that, under the efficient allocation, each private firm uses more capital and labor than each SOE. However, because the production function has decreasing returns to scale, both types of firms still use positive amounts of capital and labor.

Combining the relationship $k_P^E = \rho k_S^E$ with the aggregate resource constraint, I obtain the efficient capital and labor allocations:

$$\begin{aligned} k_P^E &= \frac{\rho \bar{K}}{s\rho + (1-s)}, & k_S^E &= \frac{\bar{K}}{s\rho + (1-s)}, \\ l_P^E &= \frac{\rho}{s\rho + (1-s)}, & l_S^E &= \frac{1}{s\rho + (1-s)}. \end{aligned}$$

Aggregate output under the efficient allocation is therefore

$$Y^E = Az_S \bar{K}^\alpha [s\rho + (1-s)]^{1-\alpha-\beta}.$$

This represents the highest aggregate output that can be achieved using the economy's fixed supplies of capital and labor when firm-specific financing constraints do not prevent the social planner from reallocating resources.

4 Theoretical results

The previous section derives the capital and labor allocations under the decentralized equilibrium and the efficient allocation, as well as the aggregate output corresponding to each allocation. Based on these results, this section compares the two types of allocation. First, I analyze capital misallocation caused by asymmetric financing constraints through differences in the marginal product of capital. Then, I compare aggregate output under the decentralized equilibrium with aggregate output under the efficient allocation, so that I can calculate the total output loss caused by capital misallocation. Finally, I introduce a

negative productivity shock and analyze how a further tightening of private firms' financing constraints amplifies the decline in aggregate output.

4.1 Capital Misallocation

In the decentralized market equilibrium, the private-firm financing constraint binds while the SOE constraint is slack. Therefore,

$$MPK_P^D - MPK_S^D = \mu_P^D > 0. \quad (5)$$

This result means that one additional unit of capital allocated to private firms would produce more output than the same unit allocated to SOEs. However, private firms cannot obtain more capital because they have already reached their financing limit.

In the efficient allocation, the social planner reallocates capital until the two firm types have the same marginal product of capital, as stated in Proposition 2.

Because the production function shows a diminishing marginal product of capital, the marginal product of capital decreases as a firm uses more capital. In the decentralized equilibrium, private firms have a higher marginal product of capital than SOEs. This indicates that transferring capital from SOEs to private firms can increase aggregate output. The social planner will continue this reallocation until the marginal products of capital are equal.

During this process, private firms' capital usage increases, while SOEs' capital usage decreases. Since the decentralized equilibrium and the efficient allocation have the same aggregate capital supply, private firms receive too little capital in the decentralized equilibrium, while SOEs receive too much capital, relative to the efficient allocation. Thus,

$$k_P^D < k_P^E, \quad k_S^D > k_S^E.$$

This is the capital misallocation studied in this paper. The more productive private firms use too little capital, while the less productive SOEs use too much capital.

Since both allocations satisfy the capital resource constraint in Equation (1), subtracting the two versions of the condition gives

$$s(k_P^E - k_P^D) = (1 - s)(k_S^D - k_S^E).$$

The left-hand side is the amount of capital missing from the private sector. The right-hand side is the excess amount of capital allocated to the SOE sector. Therefore, the capital missing from private firms is exactly equal to the excess capital used by SOEs.

4.2 Aggregate Output Loss

This subsection compares aggregate output under the decentralized allocation and the efficient allocation. The two allocations have the same aggregate capital supply $\bar{K}$, the same labor supply, and the same aggregate productivity level A . Therefore, the difference in output is only caused by the different allocation of capital and labor.

To explain why $Y^E > Y^D$, consider reallocating a small amount of aggregate capital, denoted by $dK > 0$, from the SOE sector to the private sector.

Therefore, using Equation (5), the total change in aggregate output is

$$dY = \mu_P^D dK > 0.$$

This result means that aggregate output can still be increased through capital reallocation. Therefore, the decentralized allocation is not efficient. Since the efficient allocation

maximizes aggregate output under the same resource constraints, the baseline case gives

$$Y^E > Y^D.$$

The total output loss caused by capital misallocation is defined as

$$\text{Loss}^Y = Y^E - Y^D > 0.$$

I can also measure the output loss as a proportion of the pre-shock efficient output:

$$\text{Relative Output Loss} = \frac{Y_0^E - Y_0^D}{Y_0^E}.$$

This output loss is not caused by a difference in the total amount of capital or labor. Instead, it is caused by the inefficient allocation of these resources. Financial constraints prevent more productive private firms from gaining enough capital. As a result, the same amount of resources produces less aggregate output in the decentralized equilibrium.

4.3 Aggregate Productivity Shock and Amplification

This section analyzes the effect of a negative aggregate productivity shock on output. Beyond the direct productivity effect, the model introduces a financing-response effect: lower productivity reduces private firms' borrowing capacity, tightening their credit constraints. Because private firms are more productive, reallocating capital away from them induces an additional drop in aggregate output—the amplification effect. Comparing economies with and without this financial response isolates the direct effect from the amplification effect.

Consider a negative aggregate productivity shock. Although the model is static, subscripts $t = 0$ and $t = 1$ denote the baseline and post-shock equilibrium states, respectively,

rather than time-series dynamics. Aggregate productivity declines from A_0 to

$$A_1 = (1 - \varepsilon)A_0, \quad 0 < \varepsilon < 1,$$

where ε parameterizes the shock magnitude.

4.3.1 Direct Productivity Effect

If financing capacities remain fixed after the shock, equilibrium factor allocations are unaffected. Because the shock reduces the productivity of private firms and SOEs by the same proportion, relative productivities and borrowing constraints remain unchanged. Pre-shock factor allocations continue to clear factor markets, causing aggregate output to decline exactly proportionally:

$$Y_1^{D,\text{fixed}} = (1 - \varepsilon)Y_0^D. \quad (6)$$

Equation (6) implies a direct output reduction of εY_0^D , which isolates the direct productivity effect without financial feedback.

4.3.2 Financial Responses

In the absence of a distinct banking sector, I model financial feedback via a reduced-form specification.

The financial response follows Assumption 2: private-firm financing capacity declines after the shock, while SOE financing capacity remains unchanged.

The parameter ϕ_P measures the sensitivity of private borrowing capacity to productivity shocks. Because $\varepsilon > 0$ and $\phi_P > 0$, the shock lowers $\lambda_{P1} < \lambda_{P0}$, tightening private borrowing constraints. This forces private firms to reduce capital usage, worsening capital misallocation and deepening the output decline.

To ensure economic feasibility, the post-shock private-firm financing multiple must sat-

isfy $\lambda_{P1} \geq 1$. Under Assumption 2, this requires

$$\lambda_{P1} = (1 - \phi_P \varepsilon) \lambda_{P0} \geq 1,$$

which implies

$$\phi_P \varepsilon \leq 1 - \frac{1}{\lambda_{P0}}.$$

This condition ensures that the post-shock private-firm financing limit, $\lambda_{P1} a_P$, remains at or above the firm's net worth, a_P .

To isolate the financial-capacity channel captured by λ_g , I assume that firm net worth and aggregate capital supply remain constant across states ($a_{g1} = a_{g0} = a_g$ and $\bar{K}_1 = \bar{K}_0 = \bar{K}$). I maintain the binding private-firm condition in Equation (2) in both states, with $\mu_{Pj}^D > 0$ for $j \in \{0, 1\}$, whereas SOE financing constraints remain non-binding after the shock.

4.3.3 Amplification Effect

Before the shock. The binding private-firm condition in Equation (2), evaluated before the shock, and capital-market clearing in Equation (1) determine SOE capital:

$$k_{S0}^D = \frac{\bar{K} - s \lambda_{P0} a_P}{1 - s}.$$

Baseline aggregate output Y_0^D is given by Equation (4), evaluated at $A = A_0$ and the pre-shock capital allocation.

After the shock without a financial response. When $\lambda_{P1} = \lambda_{P0}$, factor allocations remain constant, so aggregate output is given by Equation (6).

After the shock with a financial response. Under Assumption 2, private capital usage

falls to:

$$k_{P1}^{D,\text{tight}} = (1 - \phi_P \varepsilon) \lambda_{P0} a_P.$$

By capital-market clearing, SOE capital expands to:

$$k_{S1}^{D,\text{tight}} = \frac{\bar{K} - s k_{P1}^{D,\text{tight}}}{1 - s}.$$

Define $q_{P1}^{D,\text{tight}}$ and $q_{S1}^{D,\text{tight}}$ analogously to q_P^D and q_S^D in Equation (3), using the post-shock capital allocations. Post-shock output under financial tightening is:

$$Y_1^{D,\text{tight}} = A_1 \left[s q_{P1}^{D,\text{tight}} + (1 - s) q_{S1}^{D,\text{tight}} \right]^{1-\beta}.$$

Given the maintained binding configuration, the absolute and relative amplification effects are defined as:

$$\text{Amplification Loss} = Y_1^{D,\text{fixed}} - Y_1^{D,\text{tight}} > 0. \quad (7)$$

The relative amplification loss scales Equation (7) by pre-shock efficient output:

$$\text{Relative Amplification Loss} = \frac{\text{Amplification Loss}}{Y_0^E}.$$

5 Calibration

5.1 Calibration Design

This section quantifies the three main results of the model through numerical calibration. Calibration does not estimate the parameters of the model. Instead, it assigns reasonable values to the parameters and examines whether the model can produce the capital misal-

location and output loss predicted by the theoretical analysis. Specifically, I first assign baseline parameter values to the economy and calculate the decentralized and efficient allocations. I then introduce a negative productivity shock and its effect on financing capacity to calculate the amplification effect caused by the financial constraint.

5.1.1 Parameter Assignment

Table 1 reports the benchmark parameter values. The aggregate productivity level A_0 and SOE productivity z_S are normalized to one. The remaining values are based on empirical proxies where available or assigned to satisfy the model's assumptions and baseline equilibrium configuration. Appendix B provides the detailed sources and calculations. Because ϕ_P governs the strength of the amplification mechanism, I also report results across alternative values below.

Table 1: Benchmark Parameter Values

Parameter	Value	Description
A_0	1.0000	Initial aggregate productivity
z_S	1.0000	Productivity of SOEs
z_P/z_S	1.3600	Relative productivity of private firms
α	0.42441	Capital-output elasticity
β	0.47859	Labor-output elasticity
s	0.6200	Private-firm observation share (proxy)
a_P	1.0000	Net worth of private firms
a_S	0.5620	Net worth of SOEs
λ_P	2.3800	Leverage-based proxy for private firms' financing multiple
λ_S	4.3900	Leverage-based proxy for SOEs' financing multiple
$\bar{K}$	2.3300	Aggregate capital supply
$\bar{L}$	1.0000	Aggregate labor supply
ε	0.1000	Size of the productivity shock
ϕ_P	0.5000	Sensitivity of private firms' financing capacity

5.2 Benchmark Allocation and Output Loss

Table 2 compares the decentralized and efficient allocations using the same aggregate capital supply, $\bar{K} = 2.33$, and labor supply, $\bar{L} = 1$. The decentralized allocation reflects firm-specific financing constraints, whereas the efficient allocation maximizes output subject only to the aggregate resource constraints.

Table 2: Decentralized and efficient allocations prior to the shock

Outcome	Decentralized	Efficient
Private capital	2.3800	3.6637
SOE capital	2.2484	0.1539
MPK gap	0.1693	0.0000
Aggregate output	1.7732	1.8637

The baseline equilibrium has the assumed binding configuration. Private-firm capital equals its maximum financing amount, $k_P^D = \lambda_P a_P = 2.38$, whereas SOE capital remains below its financing limit, $k_S^D = 2.2484 < \lambda_S a_S \approx 2.467$. In comparison, under the efficient allocation, private-firm capital increases to 3.6637 and SOE capital decreases to 0.1539. Thus, financing constraints prevent capital moving from less productive SOEs to more productive private firms.

This capital misallocation is also shown in the difference in the marginal product of capital between the two types of firms. In the decentralized equilibrium, the MPK gap is 0.1693, while the MPKs are the same under the efficient allocation. Capital misallocation, therefore, creates an aggregate output loss. As reported in Table 2, the resulting pre-shock output loss is 0.0906, corresponding to 4.86% of efficient output. Thus, even before the negative shock, differences in financing capacity generate capital misallocation and reduce aggregate output.

5.3 Financing Capacity and Capital Misallocation

To further evaluate how financing disparity affects capital misallocation, I vary the ratio λ_S/λ_P by changing λ_P , while holding $\lambda_S = 4.39$ and the other baseline parameters fixed. A higher ratio means that private firms face a tighter financing constraint relative to that of SOEs. It lowers private firms' financing ceiling, causes more capital to be allocated to less productive SOEs, and widens the MPK gap.

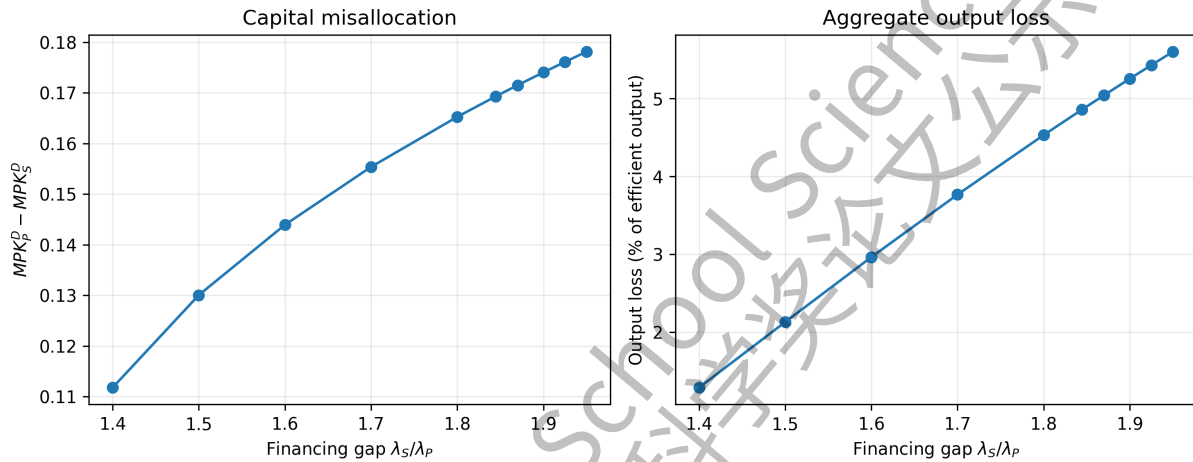


Figure 1: Financing-multiple ratio, Capital Misallocation, and Aggregate Output Loss

Table 3: Financing gap and misallocation loss rate

Financing gap λ_S/λ_P	λ_P	MPK gap	Misallocation loss rate
1.400	3.136	0.1118	1.29%
1.500	2.927	0.1301	2.13%
1.600	2.744	0.1439	2.96%
1.700	2.582	0.1554	3.77%
1.800	2.439	0.1652	4.53%
1.845	2.380	0.1693	4.86%
1.870	2.348	0.1715	5.04%
1.900	2.311	0.1740	5.26%
1.925	2.281	0.1761	5.43%
1.950	2.251	0.1781	5.60%

Figure 1 and Table 3 show that increasing λ_S/λ_P from 1.4 to 1.95 raises the MPK gap from 0.1118 to 0.1781 and the relative output loss from 1.29% to 5.60%. At the benchmark

ratio, $\lambda_S/\lambda_P = 4.39/2.38 \approx 1.84$, the relative output loss is 4.86%. When the financing capacity gap is lower than its baseline value, private firms can obtain more capital, reducing both the MPK gap and the output loss. Conversely, when the financing capacity gap is higher than the baseline value, private firms' financing constraints further tighten, and capital misallocation and output loss become more severe.

These results show that capital misallocation depends not only on productivity differences across firms but also on differences in financing capacity. When more productive private firms face tighter financing constraints, a larger financing capacity gap leaves more capital with less productive SOEs, thereby increasing the gap between decentralized and efficient aggregate output.

5.4 Financial Response and Amplification Effects

To analyze how negative shocks amplify total output loss through financing constraints, I decompose the post-shock total output loss into three parts: pre-existing capital misallocation, the direct productivity loss, and the financial amplification caused by the decrease in private firms' financing capacity.

Before the negative shock occurs, decentralized and efficient outputs are 1.7732 and 1.8637, implying a misallocation loss of 0.0906, or 4.86% of the efficient output.

Under the benchmark shock $\varepsilon = 0.10$, holding financing capacity fixed, the direct loss is 0.1773, or 9.51% of the pre-shock efficient output. This loss only comes from the decrease in productivity and does not involve the further effects of financing constraints.

Allowing financing capacity to respond reduces the private-firm financing multiple from $\lambda_{P0} = 2.38$ to $\lambda_{P1} = (1 - \phi_P \varepsilon) \lambda_{P0} = 2.261$. The decrease in financing capacity further reduces the amount of capital that private firms can obtain, and the financial amplification loss is 0.0115, which is equal to 0.62% of the pre-shock efficient output.

As shown in Table 4, the three components sum to a total loss of 0.2794, or 14.99% of

Table 4: Decomposition of Aggregate Output Loss

Component	Output loss	Relative loss
Misallocation before shock	0.0906	4.86%
Direct productivity shock	0.1773	9.51%
Financial amplification	0.0115	0.62%
Total loss	0.2794	14.99%

pre-shock efficient output.

To further investigate how the financial response affects the amplification effect, I vary ϕ_P while holding the productivity shock and all other baseline parameters fixed. A larger ϕ_P produces a greater decline in λ_{P1} . Thus, the financial amplification loss and total output loss both increase.

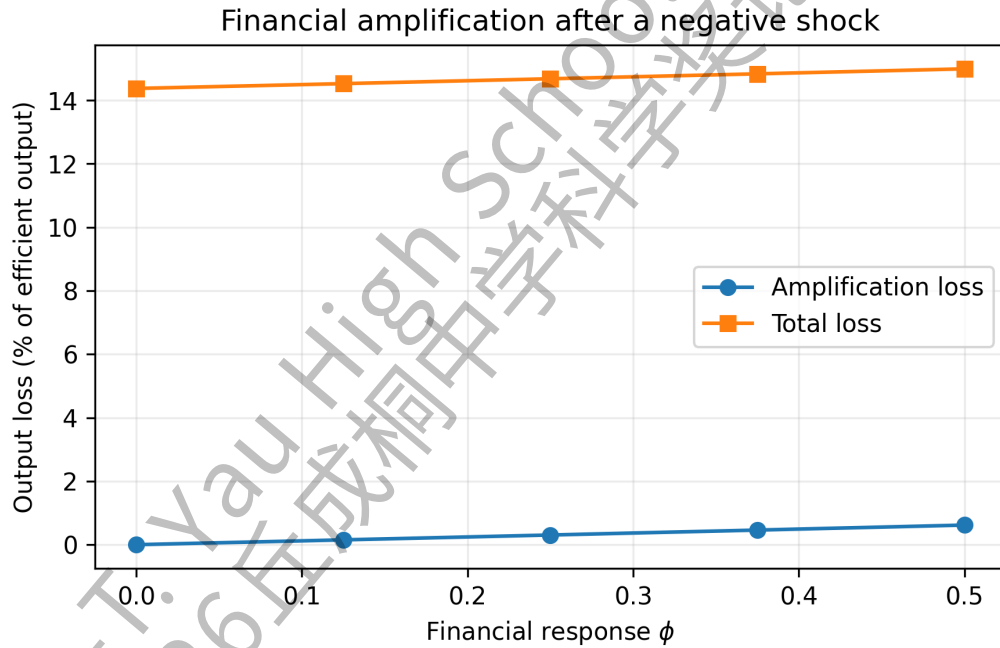


Figure 2: Financial Amplification after a Negative Shock

The blue line in Figure 2 represents the financial amplification loss. When $\phi_P = 0$, the negative shock does not affect private firms' financing capacity, so the financial amplification effect is zero. As ϕ_P increases, private firms' financing capacity becomes more sensitive to the negative shock, so the financial amplification loss continuously increases.

At the benchmark value $\phi_P = 0.5$, the financial amplification loss reaches 0.62% of the pre-shock efficient output, while the total output loss reaches 14.99%. Thus, a stronger financial response further widens the financing capacity gap, shifts capital away from high-productivity private firms, and amplifies the effect of the initial productivity shock.

5.5 Robustness Checks

To test whether the main conclusion depends on the benchmark calibration, I change the productivity disparity between private firms and SOEs, capital-output elasticity, labor-output elasticity, aggregate capital supply, and the private-firm share one at a time, holding all other parameters at their benchmark values.

Table 5: Robustness Results

Experiment	Parameter	Value	MPK gap	Misallocation loss (%)	Direct shock loss (%)	Amplification loss (%)	Total loss (%)
Productivity ratio	z_P/z_S	1.2000	0.0947	2.2221	9.7778	0.3946	12.3945
	z_P/z_S	1.3600	0.1693	4.8594	9.5141	0.6176	14.9911
	z_P/z_S	1.5000	0.2322	6.8174	9.3183	0.7661	16.9018
Capital elasticity	α	0.3760	0.1427	3.4332	9.6567	0.5564	13.6463
	α	0.4244	0.1693	4.8594	9.5141	0.6176	14.9911
	α	0.4470	0.1824	5.7020	9.4298	0.6442	15.7760
Labor elasticity	β	0.4240	0.1532	3.4454	9.6555	0.5747	13.6756
	β	0.4786	0.1693	4.8594	9.5141	0.6176	14.9911
	β	0.5040	0.1778	5.7289	9.4271	0.6389	15.7950
Aggregate Capital Supply	$\bar{K}$	2.1000	0.1606	3.4691	9.6531	0.6163	13.7385
	$\bar{K}$	2.2000	0.1650	4.0900	9.5910	0.6187	14.2997
	$\bar{K}$	2.3300	0.1693	4.8594	9.5141	0.6176	14.9911
Share of Private Firm	s	0.5500	0.1728	5.7471	9.4253	0.5639	15.7362
	s	0.6200	0.1693	4.8594	9.5141	0.6176	14.9911
	s	0.6500	0.1678	4.4659	9.5534	0.6400	14.6592

Notes: For each scenario, all relative losses are expressed as percentages of the corresponding pre-shock efficient output, Y_0^E .

The comparative statics support the model's mechanism. Increasing the productivity ratio z_P/z_S from 1.20 to 1.50 raises the MPK gap from 0.0947 to 0.2322 and the pre-shock misallocation loss from 2.22% to 6.82%. This indicates that when the productivity gap

between private firms and SOEs increases, allocating capital to lower-productivity SOEs causes an even larger efficiency loss.

Increasing $\bar{K}$ from 2.10 to 2.33 also raises the MPK gap from 0.1606 to 0.1693 and misallocation loss from 3.4691% to 4.8594%. This is because the private-firm constraint remains binding. When the aggregate capital supply increases, private firms cannot obtain the amount of capital that exceeds their financing limit, and more capital is allocated to lower-productivity SOEs. As a result, the MPK gap and output loss increase.

The model also generates positive MPK gaps and output losses throughout the reported ranges of α and β . In contrast, increasing the private-firm share from 0.55 to 0.65 reduces the pre-shock misallocation loss from 5.75% to 4.47%, because lower-productivity SOEs account for a smaller share of aggregate production. Overall, the model's qualitative conclusions remain unchanged across the parameter ranges reported in Table 5.

6 Conclusion

This paper investigates how the disparity in financing constraints between China's SOEs and private firms affects capital allocation and total output, and whether a further tightening of private firms' financing constraints amplifies the effect of negative productivity shocks. To answer these questions, this paper establishes a static model that includes private firms and SOEs and compares the decentralized equilibrium under financing constraints with the efficient allocation, which is not constrained by firms' financing capacities. This paper uses the MPK gap between the two types of firms to measure capital misallocation and divides the post-shock output loss into the pre-shock capital misallocation loss, the direct productivity loss, and the financial amplification loss.

The baseline result shows that despite private firms having higher productivity, their financing constraints limit the amount of capital they can obtain, while SOEs with lower

productivity obtain too much capital. This leads to an MPK gap and causes decentralized output to be 4.86% lower than efficient output. A 10% negative productivity shock further produces a direct output loss of 9.51%, while the decrease in private firms' financing capacity produces an additional amplification loss of 0.62%, causing the total output loss to reach 14.99%. The comparative statics and robustness checks also show that when the financing capacity disparity becomes larger, capital misallocation and output loss worsen.

These results show that the effectiveness of credit policy depends not only on the amount of credit but also on how credit is allocated among different firms. Credit expansion can increase investment, but if the additional capital mainly flows to low-productivity SOEs, its contribution to total output will be limited. Thus, during economic downturns, preventing private firms' financing constraints from becoming excessively tight may help reduce capital misallocation and the additional output loss caused by financing constraints.

This paper uses a simple static model with two types of firms, and some parameters come from different studies or are set as scenario parameters. Thus, the model should be understood as a quantitative illustration of the relationship between financing disparity and capital misallocation rather than a precise causal estimate. Future research can use more complete firm-level data to estimate the relevant parameters directly and develop a dynamic model to further analyze firms' net worth accumulation, banks' credit decisions, and long-term capital reallocation.

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productivity and marginal products of capital. After drafting my basic model, I discussed the model with Liao Jie. She suggested that I further investigate how a negative productivity shock affects or amplifies the initial loss. After considering this suggestion, I introduced a negative productivity shock into the model and compared the pre-shock economic results with the post-shock economic results. I first kept firms' financing capacities unchanged so that I could calculate the loss caused by the direct productivity shock. Then, I allowed private firms' financing capacity to decrease further and calculated the additional financial amplification loss. Through this comparison, I divided the total output loss into the initial capital misallocation loss, the direct productivity loss, and the amplification loss. During my research, I faced several difficulties. The first was the lack of firm-level data that could directly correspond to all the parameters in the model. Liao Jie suggested that I use a parameter calibration method instead of trying to estimate the entire model econometrically. During the calibration process, I also asked Peng Ke for advice on how to measure the parameters. For example, when the parameters could not be directly observed, she suggested that I find appropriate proxy variables. After that, I started to search for relevant papers and publicly available data sources and transformed the information from the references and data into parameters that fit the model. I also mentioned the source and calculation method of every parameter, as well as the specific methods used for parameters that I could not estimate directly. My second difficulty was that I was initially unfamiliar with using code to complete parameter calibration and numerical analysis. Liao Jie guided me in learning how to use Python to write code and also introduced me to how model equations could be transformed into numerical calculations. After understanding the basic method, I independently wrote and adjusted the calculation procedure, checked the equilibrium conditions, and obtained the final numerical results. I also independently created the tables and graphs in the paper and completed the comparative statics and robustness checks. The division of responsibilities between the two advisers and me was as follows: Peng Ke helped

me develop the initial research direction, provided suggestions on my theoretical analysis and selection of proxy variables, and read different versions of my paper, pointing out areas that needed further revision. Liao Jie suggested that I add an amplification mechanism and use a calibration method to address the lack of data, and guided me in learning Python so that I could complete the numerical calculations. I independently completed the literature search and reading, specific model construction and derivation, parameter search and calculation, numerical analysis, equilibrium condition checking, graph production, result interpretation, and final paper writing and editing. This research did not involve any other students or team members.

AI Usage: From July to September 2026, I used DeepSeek to assist me with my research. The main purpose of using AI was to help me check my research methods and the applicability of the economic mechanisms, and also to make sure that the way I transformed economic concepts into mathematical expressions and numerical calculations was correct. I also used AI to assist me in finding relevant keywords, potentially relevant papers and resources, explaining economic and mathematical concepts that I was unfamiliar with, checking parts of the derivation process and numerical results, and correcting my English grammar. DeepSeek was not directly involved in determining this research's question, final model, parameter choices, calculation methods, or research results. When AI provided relevant references, mathematical suggestions, or explanations of the results, I checked the original sources, recalculated the numbers, and determined whether the relevant recommendations were correct based on the model assumptions and equilibrium conditions. I decided whether to adopt the suggestions provided by AI. The final model, analysis, conclusions, and wording of the paper were all independently completed by me.

Appendix A Derivation of Firm Optimality Conditions

The financing constraint can be written as

$$\lambda_g a_g - k_g \geq 0. \quad (8)$$

Let $\mu_g \geq 0$ denote the multiplier associated with this constraint. The firm's Lagrangian function is

$$\mathcal{L}_g = Az_g k_g^\alpha l_g^\beta - w l_g - r k_g + \mu_g (\lambda_g a_g - k_g). \quad (9)$$

Differentiating with respect to labor gives

$$\frac{\partial \mathcal{L}_g}{\partial l_g} = \beta Az_g k_g^\alpha l_g^{\beta-1} - w = 0, \quad (10)$$

$$MPL_g = \beta Az_g k_g^\alpha l_g^{\beta-1} = w. \quad (11)$$

The conditional labor demand is

$$l_g^* = \left(\frac{\beta Az_g (k_g^*)^\alpha}{w} \right)^{\frac{1}{1-\beta}}. \quad (12)$$

Differentiating the Lagrangian with respect to capital gives

$$\frac{\partial \mathcal{L}_g}{\partial k_g} = \alpha Az_g k_g^{\alpha-1} l_g^\beta - r - \mu_g = 0, \quad (13)$$

$$MPK_g = \alpha Az_g k_g^{\alpha-1} l_g^\beta = r + \mu_g. \quad (14)$$

The conditional capital demand is

$$k_g^* = \left(\frac{\alpha Az_g (l_g^*)^\beta}{r + \mu_g} \right)^{\frac{1}{1-\alpha}}. \quad (15)$$

The remaining conditions are

$$\mu_g \geq 0, \quad (16)$$

$$\lambda_g a_g - k_g \geq 0, \quad (17)$$

$$\mu_g (\lambda_g a_g - k_g) = 0. \quad (18)$$

If the financing constraint is non-binding, then

$$\lambda_g a_g - k_g > 0, \quad (19)$$

$$\mu_g = 0, \quad (20)$$

$$MPK_g = r. \quad (21)$$

If the financing constraint is binding and restricts the firm's desired capital choice, then

$$\lambda_g a_g - k_g = 0, \quad (22)$$

$$\mu_g > 0, \quad (23)$$

$$MPK_g = r + \mu_g > r. \quad (24)$$

The labor condition shows that firms equate the marginal product of labor with the wage. The capital condition shows that a binding financing constraint creates a positive wedge between the marginal product of capital and its user cost. The multiplier μ_g is the shadow value of relaxing the financing limit. A larger μ_g means that the financing constraint has a greater effect on the firm's capital choice and profit.

Appendix B Details of Parameter Calibration

This appendix provides the detailed calculations of the parameters used in the benchmark calibration.

B.1 Production Parameters

This paper normalizes the productivity of state-owned firms to 1:

$$z_S = 1. \quad (25)$$

[Jurzyk and Ruane \(2021\)](#) use log revenue productivity as the dependent variable and estimate that the coefficient for state-owned firms is -0.31 . Thus, the coefficient represents the log revenue-productivity gap between SOEs and private firms. Based on an exponential transformation,

$$\frac{z_P}{z_S} = \exp(0.31) \approx 1.36. \quad (26)$$

Thus, this paper calibrates z_P/z_S to 1.36. However, it should be noted that the method here measures revenue productivity, so this parameter should be understood as a holistic productivity disparity rather than only a technological productivity disparity.

[Tu and Shen \(2016\)](#) estimate that the average returns to scale of China's large and medium-sized industrial enterprises across 37 industries are approximately 0.903. [Jurzyk and Ruane \(2021\)](#) show that the capital share is 47%. Accordingly, this paper assigns the remaining 53% to labor. This paper assumes that the relative proportions of capital and labor in total output elasticity correspond to their reported shares. Therefore, based on the returns to scale, these two factors are:

$$\alpha = 0.903 \times 0.47 = 0.42441, \quad (27)$$

and

$$\beta = 0.903 \times 0.53 = 0.47859. \quad (28)$$

Here,

$$\alpha + \beta = 0.903 < 1, \quad (29)$$

which follows the decreasing returns to scale assumed in the model.

Jurzyk and Ruane (2021) report 25,579 private-firm observations and 15,866 SOE observations in their firm-year sample. This paper uses the share of private-firm observations as a proxy for the private-firm share:

$$s = \frac{25,579}{25,579 + 15,866} \approx 0.62, \quad (30)$$

and

$$1 - s \approx 0.38. \quad (31)$$

B.2 Financing Parameters

Li et al. (2009) define leverage as:

$$LEV_g = \frac{\text{Debt}_g}{\text{Total Assets}_g}. \quad (32)$$

Because the model's financing multiple is not directly observed, this paper uses the ratio of total assets to net worth implied by the reported leverage ratio as a leverage-based proxy for λ_g . Since net worth equals total assets minus debt, this proxy is:

$$\lambda_g = \frac{1}{1 - LEV_g}. \quad (33)$$

Thus, for private firms,

$$\lambda_P = \frac{1}{1 - 0.58} \approx 2.38, \quad (34)$$

and for SOEs,

$$\lambda_S = \frac{1}{1 - 0.772} \approx 4.39. \quad (35)$$

The ratio of their financing multiples is:

$$\frac{\lambda_S}{\lambda_P} = \frac{4.39}{2.38} \approx 1.84. \quad (36)$$

Based on these leverage-based proxies, the amount of assets per unit of net worth is 1.84 times higher for SOEs than for private firms.

B.3 Normalization and Net-Worth Calibration

This paper normalizes initial aggregate productivity, private firms' net worth, and total labor supply to 1:

$$A_0 = 1, \quad a_P = 1, \quad \bar{L} = 1. \quad (37)$$

Milhaupt (2020) reports that SOEs accounted for 38.8% of China's industrial assets in 2018. This paper therefore assigns the remaining 61.2% of industrial assets to private firms. Based on private firms' and SOEs' asset proportions, leverage ratios, and firm-year observation shares, I can calculate the average net-worth ratio between the two representative firms:

$$\frac{a_P}{a_S} = \frac{\frac{0.612(1-0.58)}{0.62}}{\frac{0.388(1-0.772)}{0.38}} \approx 1.7808. \quad (38)$$

Thus, given the normalization $a_P = 1$,

$$a_S = \frac{1}{1.7808} \approx 0.562. \quad (39)$$

This value satisfies the condition under which the SOE financing constraint is not binding:

$$k_S^D < \lambda_S a_S. \quad (40)$$

Equivalently,

$$a_S > \frac{k_S^D}{\lambda_S}. \quad (41)$$

Putting in the benchmark values, I obtain:

$$a_S > \frac{2.2484}{4.39} \approx 0.5122. \quad (42)$$

Since

$$0.562 > 0.5122, \quad (43)$$

the calibrated value satisfies the condition under which the SOE financing constraint is not binding.

B.4 Internal Calibration

The total capital supply $\bar{K}$ is internally calibrated to 2.33:

$$\bar{K} = 2.33. \quad (44)$$

This value ensures that both types of firms receive positive capital allocations while maintaining the equilibrium regime assumed in the theoretical section. Because $\bar{K}$ is internally calibrated rather than directly estimated from empirical data, Section 5.5 examines the sensitivity of the main results to alternative values of the aggregate capital supply.

B.5 Shock Parameters

This paper sets the negative shock size to:

$$\varepsilon = 0.10. \quad (45)$$

This means that productivity decreases by 10% after the shock:

$$A_1 = (1 - \varepsilon)A_0 = 0.90 \quad (46)$$

Meanwhile, this paper sets the sensitivity of private firms' financing capacity to:

$$\phi_P = 0.50. \quad (47)$$

Based on the model, the financing multiple for private firms after the shock is:

$$\lambda_{P1} = (1 - \phi_P \varepsilon) \lambda_{P0}. \quad (48)$$

Putting in the benchmark values, I obtain:

$$\lambda_{P1} = (1 - 0.50 \times 0.10) \times 2.38 = 2.261. \quad (49)$$

Thus, the financing constraint for private firms after the shock becomes:

$$k_{P1}^{D,\text{tight}} = \lambda_{P1} a_P = 2.261. \quad (50)$$

This means that the negative shock not only affects firms' output but also further tightens private firms' financing constraints and creates an additional output loss. Because ε and ϕ_P lack direct empirical estimates, I set them as scenario parameters rather than

exact empirical values. Section 5.4 examines the sensitivity of the financial amplification effect to alternative values of ϕ_P , while holding the shock size ε fixed.

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