

Name of the Participating Students: Yu

Hin Wong

High School: Shenzhen College of

International Education

Province: Guangdong

Country/Region: China

Name of Advisor(s): Siya Huang

Affiliation of Advisor(s): Shenzhen

College of International Education

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Droplets Under Electric Fields

Dynamics and Trajectory Control of Charged Evaporating Droplets Under Electric Fields

Yu Hin Wong

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Abstract

Charged droplets are more and more widely used in technologies requiring precise control of particle motion, from inkjet printers to spacecraft thrusters, so the stability and trajectory control of droplets under different influences become significant. This work aims to quantify the slip correction on droplet drag, model the stabilization and controlled trajectory of a charged droplet under an electric field, and determine how evaporation and charge-induced instability limit this control. A numerical model based on Newton's second law is developed to account for gravity, aerodynamic drag, electric forces, evaporation, and Rayleigh breakup, and it is used to assess the Knudsen slip correction across a range of pressures and diameters, derive the electric field needed to stabilize an evaporating droplet, and couple this with the Rayleigh instability limit to steer a droplet toward a target height. The results show that slip correction becomes increasingly significant for droplets with size below roughly a micrometer or at reduced ambient pressure. Because smaller droplets are more influenced by aerodynamic drag, they require greater initial velocities or later time of activation of electric field to reach the target height. Evaporation reduces the stabilizing electric field due to the d^2 -law and increases the charge ratio $\frac{qa}{q_R}$ until Rayleigh breakup, but when time is normalized by d_0^2 , droplets of all initial sizes exhibit common instability behavior and the stabilizing electric field decays collapse onto one curve when droplets approach complete evaporation. External electric fields further reduce the maximum sustainable charge ratio, especially for larger droplets. The electric field used in trajectory control is also dominated by balancing gravity rather than controlling the path itself, and evaporation, not the drag or the electric field alone, limits how long a droplet can be actively controlled before breakup occurs. These findings provide a quantitative framework for designing electric-field droplet control systems while taking into account the finite timescale imposed by evaporation and charge-induced instability.

Keywords

Aerodynamic drag; Slip correction; Charged droplets; Droplet stabilization; Electric field; Evaporation; Rayleigh Breakup; Trajectory control

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1 Introduction

A single charged droplet, negligible as it was and moved along a controlled path or held motionless in the air by nothing but an invisible electric field, is a deceptively simple phenomenon behind a wide range of technologies as different as inkjet printers and spacecraft thrusters.

Multi-physics, the use of different physics within the same context, is becoming a common strategy for innovation of many technologies. In the context of internal combustion and jet engines, for instance, electric fields have been proposed as a means of controlling lubrication [1] and droplet injection [2], providing better control than purely mechanical or aerodynamic techniques do [3, 4]. Beyond engines, there are also other established applications where charged droplets and electric fields are used: inkjet printers rely on the electrostatic deflection of extremely small droplets for precise ink placement [5], electrospray thrusters use charged droplet ejection for spacecraft propulsion and altitude control [6], and metal coating widely utilizes electrostatic spray processes [7]. Droplet stabilization experiments, in which a droplet is held constant at a target height against gravity and drag by applying a balancing electric field, are a canonical way of studying the underlying physics of these technologies [8].

All of these technologies rely on the accurate control of droplet trajectories, which requires a detailed understanding of the forces acting upon the droplet and how the physics of a charged, evaporating droplet works. Since in the most general case droplets are surrounded by an atmosphere (apart from the case of ejection in space), aerodynamic interaction is one of the most significant effect. Drag on a spherical object is typically calculated using empirical correlations, and many of such correlations that have been proposed over the years are compared systematically by Goossens [9]. However, these correlations are derived for particles with considerably large sizes compared to the mean free path of the gas; when the droplet size is small compared to the mean free path of the gas, the typical expressions for forces may not be accurate, where the continuum assumption does not apply anymore and a slip correction is necessary [10]. While this slip correction is well documented, a detailed, systematic analysis spanning the full range of pressures and diameters typically found in engineering applications is still missing from the literature. In addition, the effect of the slip correction in the presence of other external forces, in particular an electric force, has not been investigated yet.

The trajectory and stabilization of a droplet are fully described by Newton's second law. However, as the droplet travels, additional physics must be taken into account. Along the path, a liquid droplet evaporates, i.e., it loses its mass in the form of vapor released to the surrounding atmosphere. As the droplet loses its mass, the charge within the droplet becomes more and more concentrated in a small volume. This makes the repulsion forces between charges within the droplets stronger, which can lead to instability and fragmentation of the droplet. The instability of an evaporating charged droplet held in a static electric field has been investigated since the classical experiments of Doyle, Moffett, and Vonnegut [11], who found that the droplet's charge stays roughly the same while its surface shrinks. This causes the charge to become more concentrated, until the droplet eventually breaks down once it reaches the Rayleigh limit [11]. Later studies examined, confirmed, and extended this behavior using electrodynamic balances and quadrupole traps [12, 13]. However, in all of these the droplet is simply held still or oscillated in place to observe the onset of instability. No

study has investigated the significance of breakup for trajectory control. In the context of transferring physics to practical application, it is of interest to understand at which point of the trajectory instability may occur, and whether this is affected by the evaporation rate and droplet size. The effects of the electric field itself on the onset of instability must also be carefully examined. A treatment that couples evaporation-driven Rayleigh breakup with a time-varying electric field used to actively control a droplet's trajectory, where the field itself must be continuously adjusted as the droplet moves and shrinks, is what this paper investigates.

To address these limitations and gaps, the objectives of this paper are to:

1. investigate the effects of slip on the drag force and identify the conditions under which this slip becomes important;
2. investigate the problem of droplet stabilization using electric fields;
3. quantify the effects of evaporation on the stabilization of the droplet and on the control of its trajectory; and
4. study the effects of charge-induced instability, in particular the Rayleigh breakup limit, on the achievable droplet control and also the effects of the electric field on the onset of instability for a range of evaporation rates and droplet diameters of practical interest.

This is how the remaining sections of the paper are structured. Section 2 presents the theoretical physics foundations of this work, including the equations of motion, the slip correction via the Knudsen number, the d-square evaporation law, and the formulation of droplet stabilization and Rayleigh breakup, along with the numerical methodology used for the investigations. The equations derived as part of this research work and used throughout the paper are presented here. Section 3 introduces the cases investigated. Section 4 presents and discusses the results on drag and slip, evaporation's effect on the stabilizing field, Rayleigh breakup with and without an applied field, and droplet trajectories. Section 5 discusses the broader implications, and Section 6 summarizes the work's key findings and takeaways.

2 Methods

2.1 Fundamental Equations

To compute the trajectory of droplets, we develop from the Newton's second law, assuming that only three forces, gravitational force, aerodynamic drag, and Lorentz force, are acting on the droplet:

$$\vec{\mathbf{F}}_G + \vec{\mathbf{F}}_D + \vec{\mathbf{F}}_L = m_d \vec{\mathbf{a}}. \quad (1)$$

Rearranging the formula in Eq.(1), we get

$$\frac{d^2 \vec{x}_d}{dt^2} = \frac{1}{m_d} (\vec{\mathbf{F}}_G + \vec{\mathbf{F}}_D + \vec{\mathbf{F}}_L),$$

where $\vec{x}_d = (x, y)$ is the displacement of the droplet, t is the time taken, m_d is the mass of the droplet, $\vec{\mathbf{F}}_G$ is the gravitational force, $\vec{\mathbf{F}}_D$ is the drag force, and $\vec{\mathbf{F}}_L$ is the Lorentz force.

We can derive component functions from the formula:

$$\frac{d^2 x}{dt^2} = \frac{1}{m_d} (F_{L,x} + F_{D,x}) \quad (2)$$

$$\frac{d^2 y}{dt^2} = \frac{1}{m_d} (F_{L,y} + F_{D,y} - m_d g) \quad (3)$$

as gravitational force only acts in the negative y-direction.

The Lorentz force $\vec{\mathbf{F}}_L$ is defined by [14]

$$\vec{\mathbf{F}}_L = q(\vec{E} + \vec{v}_d \times \vec{B}), \quad (4)$$

but the magnetic force acting upon the droplet is negligible [3], so we take

$$\vec{\mathbf{F}}_L = q\vec{E}. \quad (5)$$

To compute the drag force, we use the formula [15]:

$$\mathbf{F}_D = \frac{1}{2} \rho_g v^2 C_D A. \quad (6)$$

Substituting the cross-sectional area of a droplet with $A = \frac{\pi d^2}{4}$, we have

$$\mathbf{F}_D = \frac{\pi}{8} \rho_g C_D d^2 v^2, \quad (7)$$

which is equivalent to

$$\vec{\mathbf{F}}_D = \frac{\pi}{8} \rho_g C_D d^2 (\vec{v}_g - \vec{v}_d) |\vec{v}_g - \vec{v}_d|, \quad (8)$$

where ρ_g is the density of the gas, d is the diameter of droplet, C_D is the drag coefficient, $(\vec{v}_g - \vec{v}_d)$ is the vector form of the velocity of gas-phase relative to the droplet, and $|\vec{v}_g - \vec{v}_d|$ is its magnitude.

We can evaluate the drag coefficient from one of the correlations [3] as follows:

$$C_D = \begin{cases} 0.424, & \text{if } Re_d > 1000, \\ \frac{24}{Re_d} \left(1 + \frac{Re_d^{\frac{2}{3}}}{6} \right), & \end{cases} \quad (9)$$

Other correlations are evaluated later.

This leads to the introduction of the Reynolds number,

$$Re_d = \frac{\rho_g d |\vec{v}_g - \vec{v}_d|}{\mu_g}, \quad (10)$$

or

$$Re_d = \frac{d |\vec{v}_g - \vec{v}_d|}{\nu_g},$$

substituting kinematic viscosity $\nu_g = \frac{\mu_g}{\rho_g}$, which is equivalent to dynamic viscosity of the gas, μ_g , divided by the gas density, ρ_g .

For $Re_d \ll 1$, $C_D \sim \frac{24}{Re_d}$.

The drag coefficient is developed for spheres that are typically large compared to the distance between molecules, which means many molecules interact with the surface. However, when the size of the droplets gets closer to the distance between molecules (mean free molecular path), the expression of drag coefficient is not valid anymore. In this case, we expect a decrease in drag because particles interact more rarely with the molecules in the air. The relative dimension between particle and the distance between molecules is given by the Knudsen number, which is defined by [10]:

$$Kn = \frac{\lambda}{d}, \quad (11)$$

where λ is the mean free molecular path (average distance between molecules in a gas) and d is the diameter of the particle.

The Knudsen number has been used for developing corrections of drag coefficient as a function of the relative dimension between particles and the mean free molecular path [10]:

$$C'_D = f_{Kn} C_D, \quad (12)$$

where

$$f_{Kn} = \frac{1}{1 + Kn[A + B \exp(-E/Kn)]}, \quad (13)$$

with Kn being the Knudsen number, and $A = 1.257$, $B = 0.4$, and $E = 1.1$ being empirically measured values from Millikan's experimental work. [10].

Applying the correction of drag coefficient from Eq.(12), the drag force from Eq.(8) becomes

$$\vec{F}_D = \frac{\pi}{8} \rho_g f_{Kn} C_D d^2 (\vec{v}_g - \vec{v}_d) |\vec{v}_g - \vec{v}_d|.$$

For droplets characterized by a small Re_d , a more compact expression can be derived by $C_D = \frac{24}{Re_d} = \frac{24\mu_g}{\rho_g d |\vec{v}_g - \vec{v}_d|}$ for $Re_d \ll 1$. Substituting C_D gives

$$\vec{F}_D = \frac{\pi}{8} \rho_g f_{Kn} \left(\frac{24\mu_g}{\rho_g d |\vec{v}_g - \vec{v}_d|} \right) d^2 |\vec{v}_g - \vec{v}_d| (\vec{v}_g - \vec{v}_d).$$

Simplifying, we have

$$\vec{F}_D = 3\pi f_{Kn} d \mu_g (\vec{v}_g - \vec{v}_d), \quad (14)$$

which is the well-known Stoke's Law with the addition of the correction factor f_{Kn} .

Substituting the correction factor from Eq.(13), we obtain the final equation to compute the drag force for low Re_d :

$$\vec{F}_D = \frac{3\pi d \mu_g (\vec{v}_g - \vec{v}_d)}{1 + Kn[A + B \exp(-E/Kn)]}. \quad (15)$$

2.2 Evaporation

Before investigating the effects of evaporation on droplets, there are two essential assumptions to be made:

1. the droplet retains a constant shape throughout evaporation;
2. the droplet is a liquid.

According to the d-square law [16], the square of a liquid droplet's diameter decreases linearly with time during the evaporation process:

$$d^2(t) = d_0^2 - Kt,$$

which can also be written as

$$\frac{d^2(t)}{d_0^2} = 1 - K \frac{t}{d_0^2}.$$

Rearranging the formula gives

$$d(t) = \sqrt{d_0^2 - Kt}, \quad (16)$$

where d_0 is the initial diameter, t is time of evaporation, and K is the evaporation rate constant. K is a mathematical function of the gas temperature and pressure and the type of liquid. The type of liquid accounts for chemical and physical properties such as volatility of the droplet itself.

2.3 Droplet Stabilization

When the droplet is stabilized by an electric force in quiescent air,

$$mg = qE_s, \quad (17)$$

the electric force opposes the gravitational force. Since the mass of the droplet is

$$m = \frac{\pi}{6} d^3 \rho_d, \quad (18)$$

substituting it gives

$$\frac{\pi}{6} d^3 \rho_d g = qE_s.$$

Rearranging the formula, we obtain electric field E_s as a function of time t :

$$E_s(t) = \frac{\frac{\pi}{6} \rho_d (d_0^2 - Kt)^{\frac{3}{2}} g}{q_d}. \quad (19)$$

The Rayleigh limit q_R , or the maximum stable charge a liquid droplet can hold, is defined by the formula [17, 18]:

$$q_R = 8\pi \left(\varepsilon_0 \sigma \frac{d^3}{8} \right)^{\frac{1}{2}}, \quad (20)$$

where σ is the surface tension of the liquid droplet and ε_0 is the electric permittivity of vacuum, $\varepsilon_0 = 8.854 \times 10^{-12} \text{F/m}$. Typically, the droplet holds a charge that is $\frac{1}{2}$ of the Rayleigh limit $q_{R,0}$ at injection conditions (which means that we use $d = d_0$):

$$q_d = \frac{1}{2} q_{R,0}. \quad (21)$$

Substituting q_d at time $t = 0$ (hence $d = d_0$) into the expression of E_s , we derive

$$E_s = \frac{\frac{\pi}{6} \rho_d (d_0^2 - Kt)^{\frac{3}{2}} g}{\frac{1}{2} \left[8\pi \left(\varepsilon_0 \sigma \frac{d_0^3}{8} \right)^{\frac{1}{2}} \right]},$$

which is equivalent to

$$E_s(t) = \frac{\rho_d g}{6\sqrt{2\varepsilon_0\sigma}} \left(d_0 - \frac{Kt}{d_0} \right)^{\frac{3}{2}} \quad (22)$$

after simplification.

2.4 Rayleigh Breakup

Two assumptions are made before investigating Rayleigh breakup:

1. the droplet is in quiescent air;
2. there are no external electric fields (This assumption is removed later).

A droplet has charges distributed on its surface. These electric charges repel each other, creating an outward electrostatic force:

$$F_E \propto \frac{q^2}{r^2}, \quad (23)$$

where r represents the radius of the droplet. Meanwhile, surface tension pulls the surface inward to retain a spherical shape:

$$F_\sigma \propto \sigma r. \quad (24)$$

Therefore, when the electric repulsion exceeds the surface tension force, the droplet becomes unstable and experiences breakup or explosion. The stability ratio is:

$$\frac{q_d}{q_R} = \begin{cases} < 1, & \text{stable droplet;} \\ = 1, & \text{Rayleigh limit / explosion,} \end{cases} \quad (25)$$

where q_d is the actual charge on the droplet and q_R is the Rayleigh limit.

2.5 Trajectory Control

To model the desired stabilization trajectory, an exponential decay to the target height is assumed:

$$y(t) = y_{\text{target}} - (y_{\text{target}} - y_0)e^{-\frac{t}{\tau}}, \quad (26)$$

where τ is a time constant. Computing the first and second derivatives with respect to t , we obtain

$$\frac{dy}{dt} = \frac{y_{\text{target}} - y_0}{\tau} e^{-\frac{t}{\tau}}, \quad (27)$$

and

$$\frac{d^2y}{dt^2} = -\frac{y_{\text{target}} - y_0}{\tau^2} e^{-\frac{t}{\tau}}. \quad (28)$$

We begin with the function of the vertical component from Eq.(3) to analyze the forces acting on the droplet. We rearrange the formula to give the electric force as the subject:

$$F_{E,y}(t) = m_d \frac{d^2y}{dt^2} + m_d g - F_{D,y}.$$

Table 1: Model Parameters and Values Used in the Numerical Simulations			
Parameter	Meaning	Units	Values for Section 3
u_b	Horizontal gas velocity	m s^{-1}	0.0
v_b	Vertical gas velocity	m s^{-1}	0.0
ρ_g	Gas density	kg m^{-3}	$\frac{p_g}{R_{\text{air}} T_g}$
$d_{p,0}$	Initial droplet diameter	m	$10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}$
q_p	Droplet charge	C	q_d
m_p	Droplet mass	kg	$\frac{\pi}{6} d_{p,0}^3 \rho_d$
ρ_d	Particle density	kg m^{-3}	1000
μ_g	Gas dynamic viscosity	$\text{Pa}\cdot\text{s}$	1.857×10^{-5}
E_s	Electric field strength	V m^{-1}	$\frac{m_p g}{q_p}$
T_g	Gas temperature	K	300
p_g	Gas pressure	Pa	101325
R_{air}	Specific gas constant for air	$\text{J kg}^{-1} \text{K}^{-1}$	287.05
t_e	Evaporation time parameter	s	30.0
f_{Kn}	Slip correction factor	—	$\frac{1}{1+\text{Kn}[1.257+0.4 \exp(-1.1/\text{Kn})]}$
y_{target}	Targeted height	m	0.25

Substituting the droplet mass from Eq.(18), the d-square law from Eq.(16), and the drag force from Eq.(7) gives:

$$F_{E,y}(t) = \frac{\pi}{6} (d_0^2 - Kt)^{\frac{3}{2}} \rho_d \left(\frac{d^2 y}{dt^2} + g \right) - \frac{\pi}{8} \rho_g C_D (d_0^2 - Kt) v^2.$$

Substituting $\frac{d^2 y}{dt^2}$ from Eq.(28), velocity of the droplet $v = \frac{dy}{dt}$ from Eq.(27), and electric force $F_E = q_d E(t)$, we express the electric field as a function of time:

$$E(t) = \frac{1}{q_d} \left[\frac{\pi}{6} (d_0^2 - Kt)^{\frac{3}{2}} \rho_d \left(-\frac{y_{\text{target}} - y_0}{\tau^2} e^{-\frac{t}{\tau}} + g \right) - \frac{\pi}{8} \rho_g C_D (d_0^2 - Kt) \left(\frac{y_{\text{target}} - y_0}{\tau} e^{-\frac{t}{\tau}} \right)^2 \right].$$

Applying the Rayleigh limit from Eq.(20) and droplet charge from Eq.(21), we have:

$$E(t) = \frac{2}{8\pi \left(\varepsilon_0 \sigma \frac{d_0^3}{8}\right)^{\frac{1}{2}}} \left[\frac{\pi}{6} (d_0^2 - Kt)^{\frac{3}{2}} \rho_d \left(-\frac{y_{\text{target}} - y_0}{\tau^2} e^{-\frac{t}{\tau}} + g \right) - \frac{\pi}{8} \rho_g C_D (d_0^2 - Kt) \left(\frac{y_{\text{target}} - y_0}{\tau} e^{-\frac{t}{\tau}} \right)^2 \right].$$

Substituting the drag coefficient from Eq.(9) for $Re_d < 1000$, Reynold's number from Eq.(10), where $v_d = \frac{dy}{dt}$ from Eq.(27) and $v_g = 0$, and the d-square law from Eq.(16), we get:

$$E(t) = \frac{2}{8\pi \left(\varepsilon_0 \sigma \frac{d_0^3}{8}\right)^{\frac{1}{2}}} \left[\frac{\pi}{6} (d_0^2 - Kt)^{\frac{3}{2}} \rho_d \left(-\frac{y_{\text{target}} - y_0}{\tau^2} e^{-\frac{t}{\tau}} + g \right) - \frac{\pi}{8} \rho_g \frac{24}{\frac{\rho_g \sqrt{d_0^2 - Kt} |0 - \frac{y_{\text{target}} - y_0}{\tau} e^{-\frac{t}{\tau}}|}{\mu_g}} \left(1 + \frac{\left(\frac{\rho_g \sqrt{d_0^2 - Kt} |0 - \frac{y_{\text{target}} - y_0}{\tau} e^{-\frac{t}{\tau}}|}{\mu_g} \right)^{\frac{2}{3}}}{6} \right) \times (d_0^2 - Kt) \left(\frac{y_{\text{target}} - y_0}{\tau} e^{-\frac{t}{\tau}} \right)^2 \right].$$

After simplification, we obtain:

$$E(t) = \frac{\sqrt{2}}{2d_0^{\frac{3}{2}} \sqrt{\varepsilon_0 \sigma}} \left[-\frac{1}{6} \rho_d (d_0^2 - Kt)^{\frac{3}{2}} \left(\frac{y_{\text{target}} - y_0}{\tau^2} e^{-\frac{t}{\tau}} - g \right) - \frac{3\mu_g |y_{\text{target}} - y_0|}{\tau} e^{-\frac{t}{\tau}} \sqrt{d_0^2 - Kt} \times \left(1 + \frac{1}{6} \left(\frac{\rho_g |y_{\text{target}} - y_0| \sqrt{d_0^2 - Kt} e^{-\frac{t}{\tau}}}{\tau \mu_g} \right)^{\frac{2}{3}} \right) \right]. \quad (29)$$

3 Investigated Cases

3.1 Droplet Stabilization in Air

Droplet stabilization can be achieved in different ways. The case considered is a droplet between two walls that is released at time zero. The droplet under gravity tends to fall. The objective is to apply an electric field and therefore an electric force to stabilize the droplet at a given height. Then, two sub-cases are considered: the first being finding the initial velocity needed to stabilize the droplet at a given height as a function of droplet diameter and the second being

```

while (rel_error > 0.00001):

    # Solving
    initial_conditions = [y_initial, ydot_initial, x_initial, xdot_initial]
    sol = odeint(solver_xydrop_EF, initial_conditions, t, args=(ub, vb, rhog, dp_0, qp, mp, visg, Es, Tg, pg, Rair, te, Correction))
    x, y, xdot, ydot = sol[:, 2], sol[:, 0], sol[:, 3], sol[:, 1]

    if (y[-1]>y_target):
        ydot_low = ydot_initial
    elif (y[-1]<y_target):
        ydot_high = ydot_initial

    rel_error = abs((y[-1]-y_target)/y_target)

    ydot_initial = (ydot_low+ydot_high)/2.0

    print("y final [m] = ", y[-1], "; error = ", rel_error, "; initial velocity [m/s]", ydot_initial)

```

Figure 1: *Python code of solver function used in investigated case 1.*

finding the time of application of the electric field needed to stabilize the droplet at a given height as a function of droplet diameter. For the latter, we assume a zero initial velocity for the droplet.

As discussed in Section 2.3, the stability condition in a quiescent medium implies that $F_{L,y} = -m_d g$. This provides a condition to find the electric field needed to stabilize the droplet, which is given by Eq.(19).

3.2 Implementation

The configurations were studied by numerically integrating the equation of motion (both along the x and y directions). The integration of the equations was performed in Python, using the `scipy.integrate.odeint` module. Because the investigation requires finding an unknown initial velocity, $\dot{y}_0$, or an unknown time of application of electric forces that allow to stabilize the droplets at a given height, y_{target} , the system is treated by adjusting at each iteration the value of $\dot{y}_0$ or time of application of the forces. A bisection root-finding algorithm is utilized to iteratively converge on the correct initial vertical velocity, or the time of application of the electric force.

The simulation computes the trajectory by solving the system of ordinary differential equations over a time vector t . Every cycle or iteration starts with these conditions:

$$[y_0, \dot{y}_0, x_0, \dot{x}_0], \quad (30)$$

which represent the initial vertical position, vertical velocity, horizontal position, and horizontal velocity respectively. During the integration process, the `odeint` function calculates the evolution of these variables over the time range t , producing the droplet trajectory. Forces along x and y , are passed directly to the `odeint` integrator, including the time of application of the electric force, for the second case.

The output solution is

x , y , $\dot{x}$, $\dot{y}$

where x and y describe the droplet position, while $\dot{x}$ and $\dot{y}$ describe the velocity components. This trajectory solver is used in this work any time the trajectory of the droplet needs to be computed. The inputs to the solver are compiled in Tables 1 and 2.

Table 2: State Variables and Period of Application

Variable	Meaning	Units	Case 1	Case 2
x	Horizontal position	m	0	0
y	Vertical position	m	0.5	0.5
$\dot{x}$	Horizontal velocity	m s^{-1}	0	0
$\dot{y}$	Vertical velocity	m s^{-1}	result of iteration	0
T	Time of application of electric force	s	—	result of iteration

For the evaluation of the configuration described in Section 3.1, the focus is on evaluating the final y position (after a long time, such that any transient is finished). The final vertical position y_{final} is used to determine whether the chosen initial velocity produces the required stabilization height.

To compute the precise initial velocity needed to stabilize the droplet at y_{target} , the algorithm executes a **while** loop that iteratively refines $\dot{y}_0$ by comparing the final droplet height with the target stabilization height:

$$\text{rel_error} = \left| \frac{y_{final} - y_{target}}{y_{target}} \right|. \quad (31)$$

If the final height is higher than the target value, the droplet has stopped too early due to drag, so the magnitude of the initial velocity must be increased (or the time of application must be reduced). If the final height is lower than the target value, then the magnitude of initial velocity must be reduced (or the time of application must be increased). The process repeats until the relative error becomes smaller than the criterion:

$$\text{rel_error} < 10^{-5}. \quad (32)$$

To find the exact injection velocity, the algorithm uses a root-finding method to narrow down the correct value:

- **Case 1: The Undershoot (Stopped Too High)**

If the droplet stops completely before reaching the target line, $y_{final} > y_{target}$, it means the initial velocity was too weak to overcome the air resistance (or the time of application was too late). The algorithm marks this velocity as the new minimum speed threshold (`ydot_low`), ensuring that the next guess results in a stronger push downward. Or equivalently, the time of application is marked as T_{max} , the maximum time.

- **Case 2: The Overshoot (Fell Too Far)**

If the droplet falls past the target line, $y_{final} < y_{target}$, it means the injection velocity was too huge. The algorithm marks this velocity as the new maximum speed threshold (`ydot_high`), ensuring that the next guess results in a softer push downward. Or equivalently, for second case, the time of application is too small, and a new minimum for that is set.

Once the new boundaries are set, the code simply takes the exact average of the new minimum and maximum limits:

$$\dot{y}_{initial} = \frac{\dot{y}_{low} + \dot{y}_{high}}{2}, \quad (33)$$

and for the another case,

$$T_{avg} = \frac{T_{min} + T_{max}}{2}. \quad (34)$$

The loop repeats this narrowing process until the droplet lands precisely on y_{target} .

4 Results

4.1 Assessment of drag force

Figure 2 shows the Knudsen factor as a function of the diameter of the particle for different ambient pressures. The values of the ambient pressure from 0.1 to 10 bar have been considered. The Knudsen factor increases with increasing diameter and approaches unity for large values of diameter. The value is very low for very small diameters. Comparison between different pressure conditions shows that lower pressure leads to lower values of the Knudsen factor at all diameters. This is consistent with the fact at lower pressures, the mean free path of gas molecules increases. When this distance between molecules becomes comparable to or larger than the particle diameter, the "slip" phenomenon becomes more evident and the Knudsen factor deviates further from unity.

Figure 3 shows the Knudsen factor as a function of the Knudsen number (Kn) for different ambient pressures. The values of the ambient pressure from 10,000 to 1,000,000 Pa have been considered. The Knudsen factor decreases with

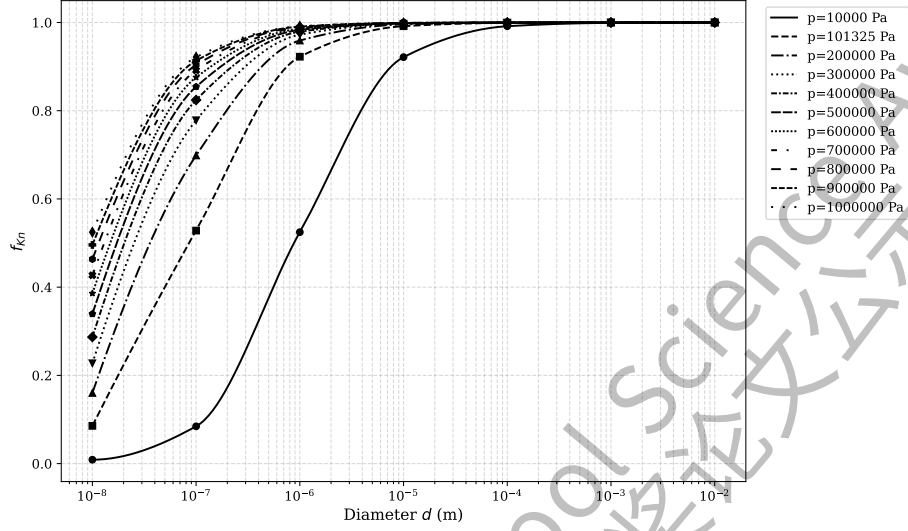


Figure 2: Knudsen factor, f_{Kn} , as a function of the particle diameter for different ambient pressures. x -axis in logarithmic scale.

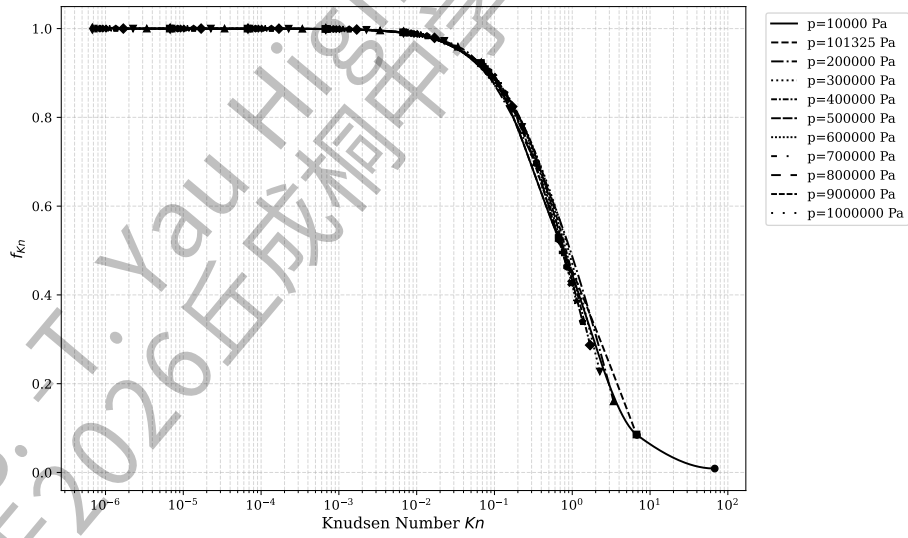


Figure 3: Knudsen factor, f_{Kn} , as a function of Knudsen number, Kn , for different ambient pressures. x -axis in logarithmic scale.

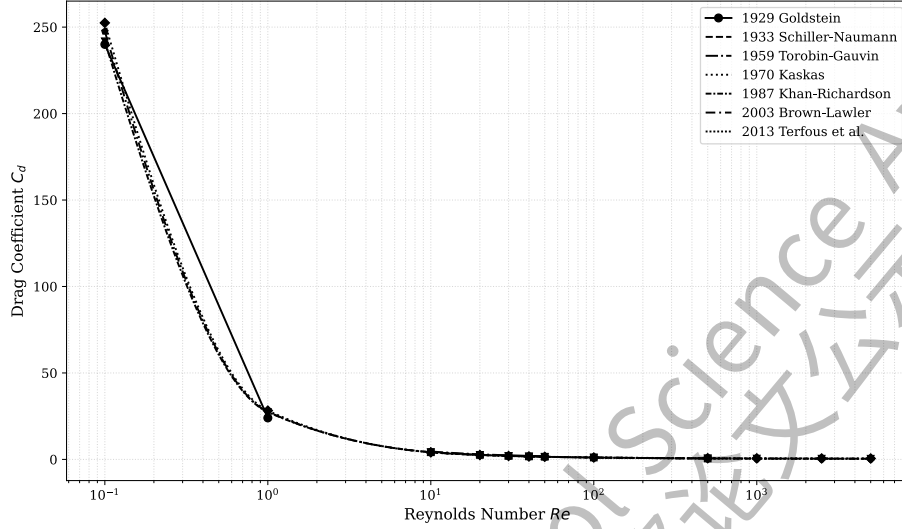


Figure 4: Drag coefficient as a function of Reynolds number for different investigators' empirical correlations. x -axis in logarithmic scale. Correlations are taken from Ref.[9].

increasing Knudsen number and approaches zero for large values of Kn . The value remains relatively constant at unity for very small Knudsen numbers. Comparison between different pressure conditions shows that lower pressure leads to slightly lower values of the Knudsen factor as the Knudsen number increases. Lower pressure slightly reduces the Knudsen factor at a constant Kn because it reduces the gas density, which enhances the "slip" effect and reduces the interaction captured by f_{Kn} . Comparing figure 3 with figure 2, we see a mirror-like behavior, which is justified by considering that Kn is inversely proportional with respect to the diameter, so Kn decreases as diameter increases.

Figure 4 shows the drag coefficient as a function of the Reynolds number for different empirical correlations developed by various investigators. The Reynolds numbers ranging from 0.1 to 5,000 have been considered. The drag coefficient decreases sharply with increasing Reynolds number for small values of the Reynolds number and approaches a relatively constant, low value for large Reynolds numbers. The value is very high for very small Reynolds numbers, exceeding 240 at $Re_d = 0.1$. Comparison between different correlations shows that while most investigators' models converge closely for Reynolds numbers greater than 1, there is a visible divergence at lower Reynolds numbers (especially with Reynolds below 1), where the 1970 Kaskas correlation predicts higher drag coefficients compared to the other investigators' models.

However, when we change the y -axis to logarithmic scale as well, we realize that

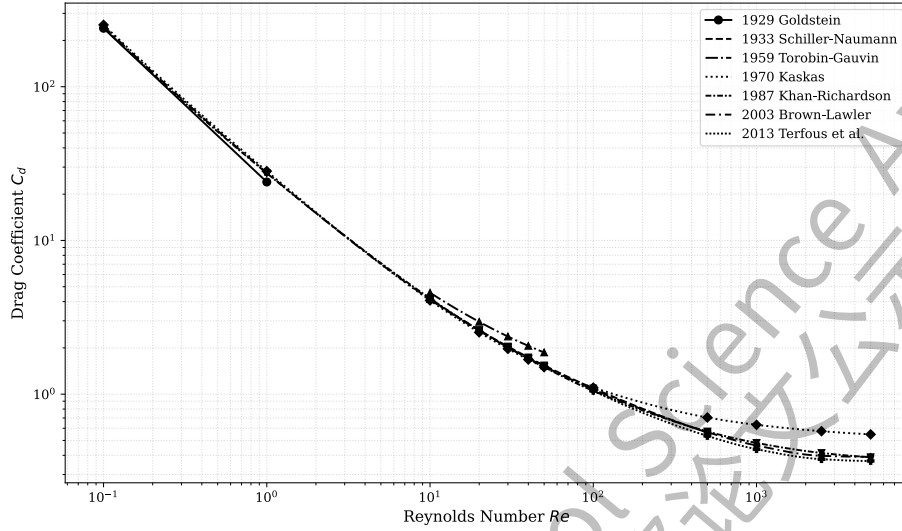


Figure 5: Drag coefficient as a function of Reynolds number for different investigators' empirical correlations. Both axes in logarithmic scale. Correlations are taken from Ref.[9].

there is instead a convergence at lower Reynolds numbers and a visible divergence at higher Reynold numbers. This is because convergence and divergence depend on the relative difference between the values rather than their absolute difference. In the linear plot (Figure 4), the large absolute values of the drag coefficient at low Reynolds numbers (where $C_D > 200$) make the correlations diverge. Conversely, at high Reynolds numbers, the values are so small that they look identical on a linear scale.

By using a plot (Figure 5) with both axes in logarithmic scale, we equalize the visual weight of these differences. We can see that at low Reynolds numbers, the models converge quite well and present strong agreement on Stokes' law. At high Reynolds numbers, however, the logarithmic scale reveals that the relative difference between the values becomes very significant that the investigators' various predictions of turbulent drag actually diverge significantly, in contrast with the situation when the values are compressed against the x-axis in the linear plot (Figure 4).

Figure 6 shows the drag force as a function of the Reynolds number for different empirical correlations developed by various investigators, assuming a particle diameter (d) of 0.1m, a kinematic viscosity of the gas (ν) of $1.8 \times 10^{-5} \text{Pa} \cdot \text{s}$, and a gas density of (ρ) of 1.184521kg/m^3 . The Reynolds numbers including 0.1 to 5,000 have been considered. The drag force increases with increasing Reynolds number and rises rapidly for values of Re_d greater than 1,000. The value is nearly zero at unity for very small Reynolds numbers (especially with

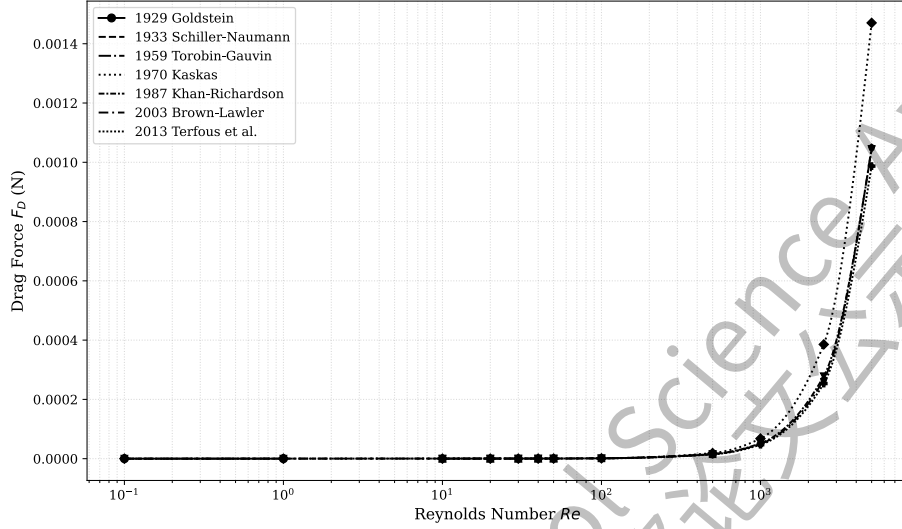


Figure 6: Drag force as a function of the Reynolds number for different investigators' empirical correlations. x -axis in logarithmic scale. Correlations are taken from Ref.[9].

Reynolds below 100). Comparison between different correlations shows that for Reynolds numbers greater than 1,000, the 1970 Kaskas correlation predicts significantly higher values of drag force compared to the other investigators' models, which are more concentrated.

4.2 Effect of evaporation on E_s

To evaluate the effect of evaporation on the stabilizing electric field, three graphs are produced assuming that the liquid droplet is made up of water, with density $\rho_d = 1000 \text{ kg/m}^3$ and surface tension $\sigma = 0.0728 \text{ N/m}$ at $\sim 20^\circ \text{C}$.

Figure 7 depicts the electric field E_s as a function of time t , where initial diameter d_0 is kept constant at 10^{-4} m and a range of values of evaporation rate constant K from 0.05×10^{-6} to $0.60 \times 10^{-6} \text{ m}^2/\text{s}$ is considered. A similar trend is evident for every curve, that E_s converges at the identical value of about 1440 V/m at $t = 0$ and declines non-linearly until it hits zero. As the evaporation rate constant increases, the curve becomes steeper, causing the electric field to reach zero faster. This phenomenon is explained by the relationship $E_s(t) \propto (d_0 - \frac{Kt}{d_0})^{\frac{3}{2}}$, where a larger K value increases the rate of reduction of droplet's size, causing the droplet to lose mass quicker and also forcing the electric field to drop to zero sooner.

Figure 8 depicts the electric field E_s as a function of normalized time $\frac{t}{d_0^2}$, where

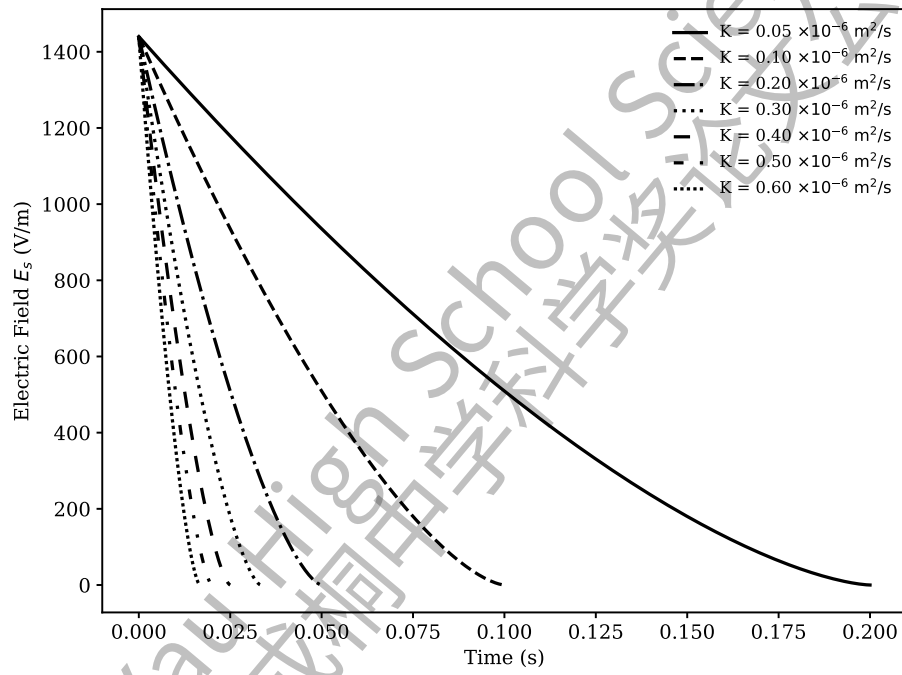


Figure 7: Electric field, E_s , as a function of time, t , for different values of evaporation rate constant, K . Initial diameter, d_0 , is kept constant at 10^{-4}m .

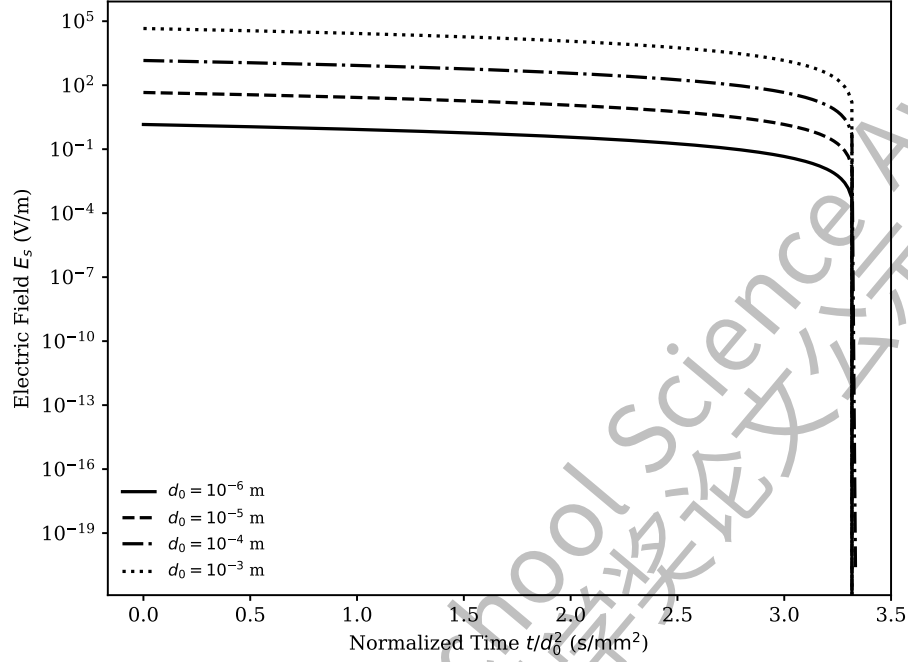


Figure 8: Electric field, E_s , as a function of normalized time, $\frac{t}{d_0^2}$, for different values of initial diameter, d_0 . Evaporation rate constant is kept at $K = 0.3 \times 10^{-6} \text{ m}^2/\text{s}$.

the evaporation rate constant is kept at $K = 0.3 \times 10^{-6} \text{ m}^2/\text{s}$ and a range of initial droplet diameter d_0 from 10^{-6} to 10^{-3} m is considered. The trend shows that larger droplets lead a higher initial electric field E_s , but the declining behavior becomes comparable when time is normalized by the square of the initial diameter. The higher initial electric field is due to the higher mass of the droplets, which is not balanced by an equal increase in droplet charge. In addition, droplet evaporation follows the d^2 -law, where the lifetime of a droplet is defined by $t_{\text{max}} = \frac{d_0^2}{K}$. Therefore, plotting against $\frac{t}{d_0^2}$ adjusts the time according to the initial droplet size, allowing the evaporation process of droplets with different diameters to be compared more fairly since bigger droplets naturally take longer to evaporate. Although larger droplets maintain a stronger electric field due to their larger initial diameter, all droplets approach complete evaporation at a similar normalized time.

Figure 9 illustrates electric field E_s as a function of initial diameter d_0 on a logarithmic scale by isolating the state of the system at time $t = 0$. A range of values of initial droplet diameter d_0 from 10^{-6} to 10^{-3} m is considered. The trend shows a straight, upward sloping line where electric field increases from

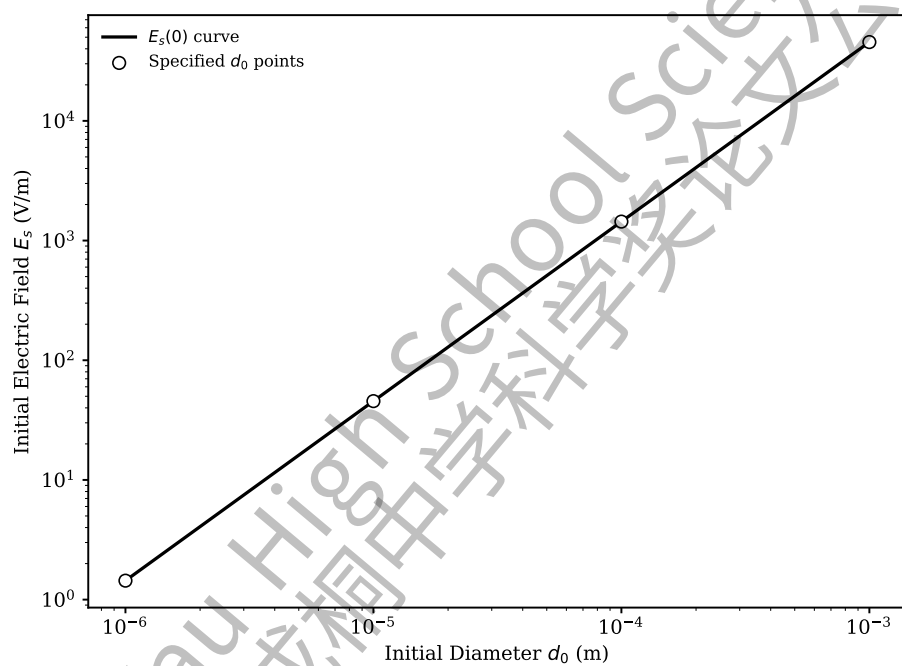


Figure 9: Electric field, E_s , as a function of initial diameter, d_0 , at time $t = 0$. Both axes in logarithmic scale.

less than 1 to more than 10^4V/m when droplet diameter increases. This is explained by

$$E_s(0) = \frac{\rho_d g}{6\sqrt{2\varepsilon_0\sigma}} \cdot d_0^{\frac{3}{2}} \quad (35)$$

when we substitute time $t = 0$ into the Eq.(22), revealing a positive relationship between electric field and initial diameter, $E_s(t) \propto d_0^{\frac{3}{2}}$. Since the axes are in logarithmic scale, the power of $\frac{3}{2}$ is transformed into a slope equal to this constant.

4.3 Effect of Rayleigh Breakup

We now investigate the stability ratio $\frac{q_d}{q_R}$ in Eq.(25).

Since the formula of q_R is

$$q_R = 8\pi \left(\varepsilon_0 \sigma \frac{d^3}{8} \right)^{\frac{1}{2}},$$

substituting the d-square law from Eq.(16) gives

$$q_R = 8\pi \left[\frac{\varepsilon_0 \sigma (d_0^2 - Kt)^{\frac{3}{2}}}{8} \right]^{\frac{1}{2}}. \quad (36)$$

With the formula of q_d being

$$q_d = \frac{1}{2} \left[8\pi \left(\frac{\varepsilon_0 \sigma d_0^3}{8} \right)^{\frac{1}{2}} \right],$$

the stability ratio $\frac{q_d}{q_R}$ becomes

$$\frac{q_d}{q_R} = \frac{\frac{1}{2} \left[8\pi \left(\frac{\varepsilon_0 \sigma d_0^3}{8} \right)^{\frac{1}{2}} \right]}{8\pi \left[\frac{\varepsilon_0 \sigma (d_0^2 - Kt)^{\frac{3}{2}}}{8} \right]^{\frac{1}{2}}},$$

and after simplification,

$$\frac{q_d}{q_R} = \frac{1}{2} \left[1 - K \left(\frac{t}{d_0^2} \right) \right]^{-\frac{3}{4}}. \quad (37)$$

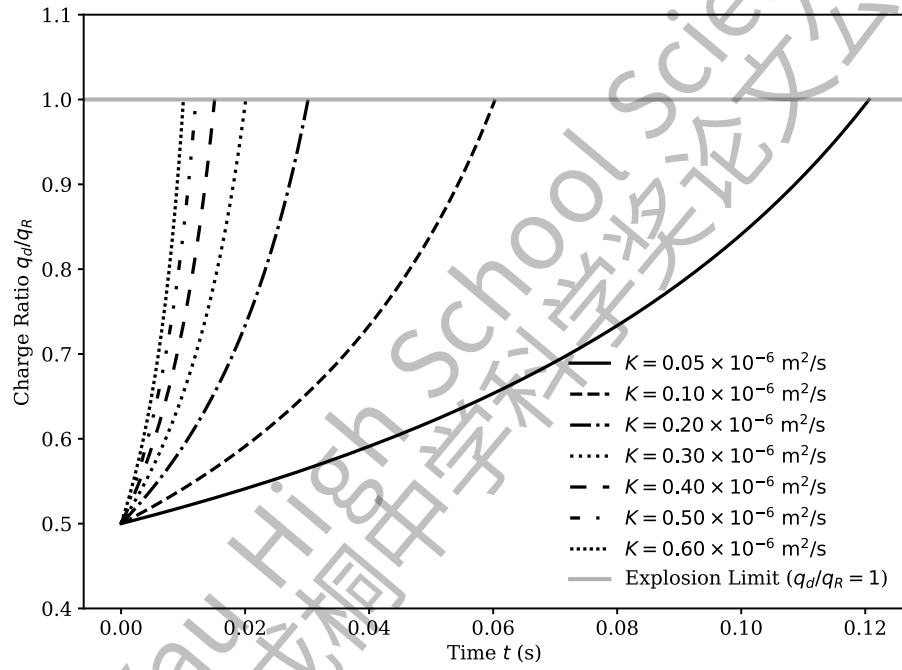


Figure 10: Charge ratio, $\frac{q_d}{q_R}$, as a function of time, t , for different values of evaporation rate constant, K . Initial diameter, d_0 , is kept constant at 10^{-4}m .

4.3.1 Without External Electric Field

We now evaluate the derived equation of charge ratio with respect to either variable K values or variable d_0 values without the effect of external electric field.

Figure 10 depicts the charge ratio $\frac{q_d}{q_R}$ as a function of time t , where initial diameter d_0 is kept constant at 10^{-4}m and a range of values of evaporation rate constant K from 0.05×10^{-6} to $0.60 \times 10^{-6}\text{m}^2/\text{s}$ is investigated. A dashed red horizontal line marks the critical “Explosion Limit” when the charge ratio is 1. The trend shows that for all values of K , the charge ratio increases exponentially over time until it hits the explosion limit, and the speed of the increase is highly dependent on the evaporation rate constant. For instance, droplets with the highest evaporation rate constant of $K = 0.60 \times 10^{-6}\text{m}^2/\text{s}$ exhibit the steepest curve, reaching the explosion limit in under 0.01 seconds, while droplets with the lowest evaporation rate constant of $K = 0.05 \times 10^{-6}\text{m}^2/\text{s}$ yield a much shallower curve that takes around 0.12 seconds to reach the same explosion limit. This trend can be explained by the Eq.(37):

$$\frac{q_d}{q_R} = \frac{1}{2} \left[1 - K \left(\frac{t}{d_0^2} \right) \right]^{-\frac{3}{4}}.$$

It is evident that as time increases, the term within the bracket $\left(1 - K \frac{t}{d_0^2} \right)$ decreases, causing the charge ratio to increase. A larger value of K makes this term decrease more rapidly, which as a result cause the charge ratio to approach 1 faster. Physically, as a droplet evaporates, its surface area and volume shrink rapidly. However, unless the charge is actively discharged, the total electrical charge on the droplet q_d remains the same. A higher evaporation rate constant causes the droplet to shrink more quickly, which makes the Rayleigh limit q_R decrease faster. As a result, the droplet reaches the explosion limit in a shorter time.

Figure 11 depicts the charge ratio $\frac{q_d}{q_R}$ as a function of normalized time $\frac{t}{d_0^2}$, where evaporation rate constant K is kept at $0.3 \times 10^{-6}\text{m}^2/\text{s}$ and a range of values of initial droplet diameter d_0 from 10^{-6} to 10^{-3}m is investigated. A dashed red horizontal line marks the critical “Explosion Limit” when the charge ratio is 1. The remarkable trend of this data is that the curves for all four droplet sizes collapse perfectly onto a single, universal trajectory that begins at an initial charge ratio of 0.5, then rising at an accelerating rate until it hits the explosion limit at a normalized time of approximately $2.0\text{s}/\text{mm}^2$. This trend is explained by the d^2 -law of evaporation in Eq.(16). By plotting the charge ratio against the normalized time $\frac{t}{d_0^2}$, the time of evaporation is adjusted according to the initial droplet size, making the evaporation timescale independent of d_0 and allowing droplets of various sizes to exhibit identical behavior. Similarly to the previous case, as $\frac{t}{d_0^2}$ increases, the term $\left(1 - K \frac{t}{d_0^2} \right)$ decreases, causing the charge ratio to

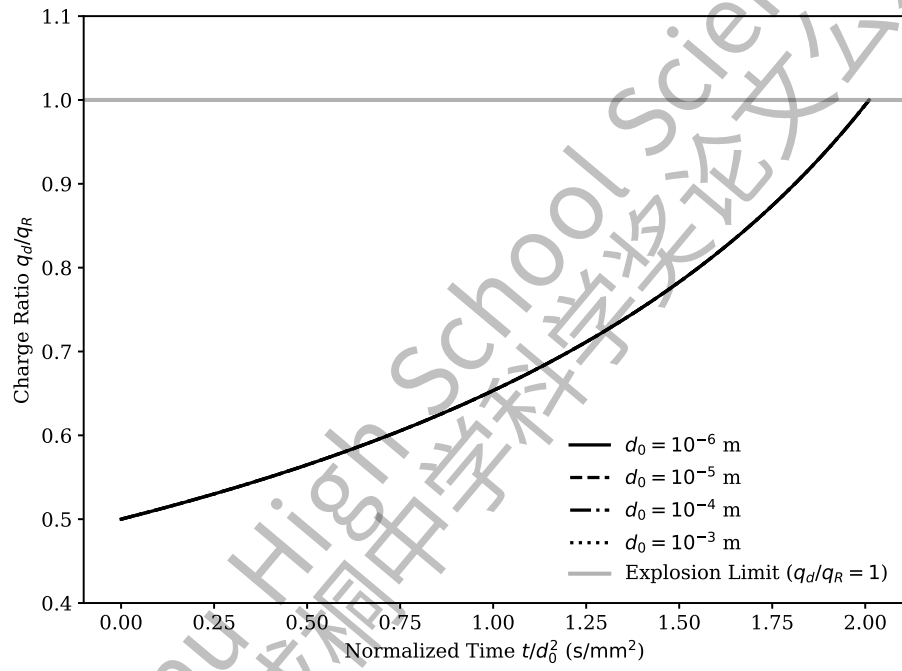


Figure 11: Charge ratio, $\frac{q_d}{q_R}$, as a function of normalized time, $\frac{t}{d_0^2}$, for different values of initial diameter, d_0 . Evaporation rate constant is kept at $K = 0.3 \times 10^{-6} \text{ m}^2/\text{s}$.

increase nonlinearly. Consequently, all droplets reach the explosion limit at the same normalized time, revealing that the evaporation process obeys the same kinetic law regardless of the droplet size when expressed in terms of normalized time.

4.3.2 With External Electric Field

We now evaluate the derived equation of charge ratio with respect to either variable K values or variable d_0 values with the effect of external electric field.

The formula of the Rayleigh limit in Eq.(20)

$$q_R = 8\pi \left(\epsilon_0 \sigma \frac{d^3}{8} \right)^{\frac{1}{2}},$$

is too conservative. In the presence of an external electric field, we break down at a much lower charge because charges are not symmetric anymore, which they accumulate preferentially at the poles of the droplet. So, locally, for the same q , there is a much higher electrostatic repulsion, which leads to earlier instabilities.

The maximum free surface charge that a dielectric droplet can sustain in an external electric field is given by[19]:

$$\frac{q_d}{q_R} = \left(\frac{\epsilon_g}{2\epsilon_g - 1} \right) \left\{ 1 - \frac{3}{2} E_0 \sqrt{\frac{r\epsilon_0}{\epsilon_g \gamma}} \left(\frac{\epsilon_d - \epsilon_g}{\epsilon_d + 2\epsilon_g} \right) \right\}, \quad (38)$$

where $\epsilon_d = 80.1$ and $\epsilon_g = 1.0$ denote the relative permittivity of drop and continuum respectively, r is the radius of the droplet, and E_0 is the electric field.

We now analyze this formula by creating two graphs to investigate the relationships between charge ratio and either diameter d or electric field E_0 with one of the two variables held constant.

Figure 12 depicts the maximum charge ratio $\left(\frac{q_d}{q_R}\right)^*$ as a function of external electric field E_0 for a fixed droplet diameter $d = 10^{-4}$ m. The maximum charge ratio represents the greatest charge ratio that the droplet can sustain before becoming unstable in the presence of an external electric field. This graph shows a linear decreasing trend, illustrating that the maximum charge ratio decreases at a constant rate as the external electric field increases. When an external electric field is not applied $E_0 = 0$, the maximum charge ratio approaches the classical Rayleigh limit $\left(\frac{q_d}{q_R}\right)^* = 1$. However, as the electric field becomes stronger, the droplet requires less total charge to undergo Rayleigh breakup, thus decreasing the charge ratio threshold. This trend can be explained directly from Eq.(38), since the electric field appears as a linear term:

$$\left(\frac{q_d}{q_R} \right)^* = A(1 - BE_0\sqrt{r}),$$

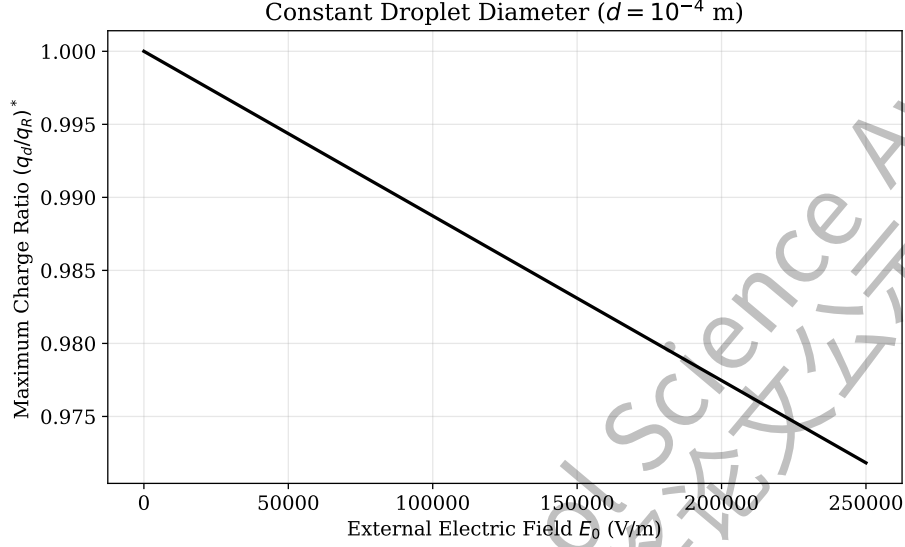


Figure 12: Maximum Charge ratio, $\frac{q_d}{q_R}$, as a function of external electric field, E_0 . Diameter, d , is kept constant at 10^{-4} m.

where $A = \frac{\epsilon_g}{2\epsilon_g - 1}$ and $B = \frac{3}{2} \sqrt{\frac{\epsilon_0}{\epsilon_g \gamma}} \left(\frac{\epsilon_d - \epsilon_g}{\epsilon_d + 2\epsilon_g} \right)$ are constants. In the case of Figure 12, the radius $r = \frac{d}{2}$ is also held constant. Because E_0 is multiplied by a constant coefficient in a negative term, each increase in electric field results in the same reduction in maximum charge ratio, which gives rise to the linear relationship. To explain in physical terms, a stronger external electric field enhances charge polarization within the droplet, causing free charges to accumulate more on opposite inner sides of the droplet. This increases the electrostatic repulsion at the droplet surface, allowing instability to occur at a ratio lower than that of the classical Rayleigh limit.

Figure 13 shows the maximum charge ratio $\left(\frac{q_d}{q_R}\right)^*$ as a function of the droplet diameter d for a fixed external electric field $E_0 = 200$ kV/m. A logarithmic scale is used for the x -axis. This graph presents a nonlinear decreasing trend, where the maximum charge ratio decreases as the droplet diameter increases. The reduction is very subtle for smaller diameters and becomes steeper as the diameter increases. This trend can be explained directly from Eq.(38), since the charge ratio depends on the square root of the droplet radius:

$$\left(\frac{q_d}{q_R}\right)^* = A(1 - BE_0\sqrt{r}),$$

where $A = \frac{\epsilon_g}{2\epsilon_g - 1}$ and $B = \frac{3}{2} \sqrt{\frac{\epsilon_0}{\epsilon_g \gamma}} \left(\frac{\epsilon_d - \epsilon_g}{\epsilon_d + 2\epsilon_g} \right)$ are constants, with $r = \frac{d}{2}$. Consequently, the maximum charge ratio varies with $\sqrt{d}$ instead of d itself, resulting

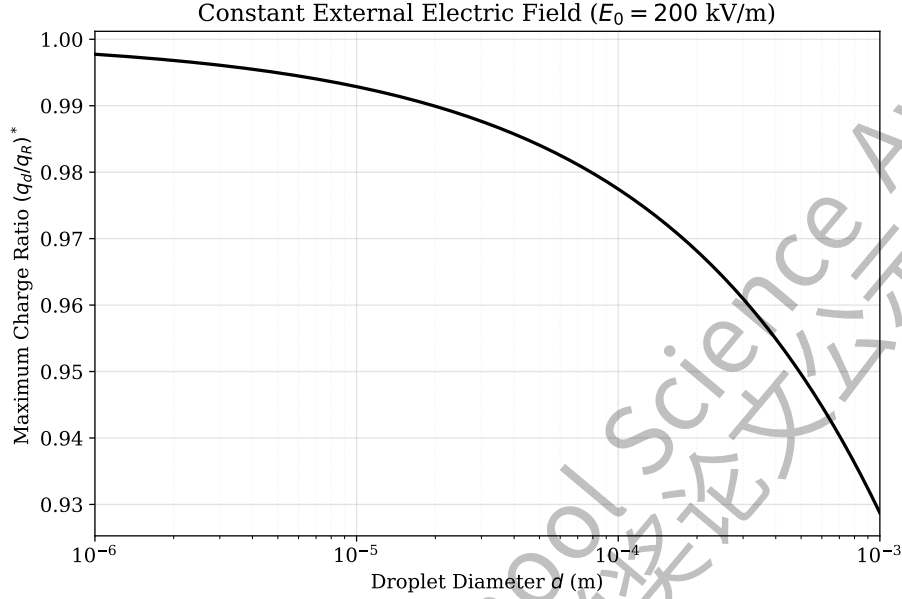


Figure 13: Maximum charge ratio, $\frac{q_d}{q_R}$, as a function of diameter, d . External electric field, E_0 , is kept constant at 200kV/m. x -axis in logarithmic scale.

in a nonlinear square-root relationship as reflected by the gradual flattening of the curve. To explain physically, larger droplets experience stronger polarization because the external electric field can displace a greater overall amount of charge across a wider span. This produces greater charge accumulation on opposite inner sides of the droplet, increasing the electrostatic stress on the surface. As a result, larger droplets become unstable at a lower ratio than that of the classical Rayleigh limit.

We now compare the evaporation-driven charge ratio (without external electric field) against the dynamic instability threshold induced by the external electric field.

For Figure 14, an evaporating droplet in equilibrium at every time is considered. This means that with increasing time, the diameter is decreasing because of evaporation and, at the same time, the applied external electric field is decreased to keep the balance with the decreased gravitational force (see Eq.(17)). From Figures 12 and 13, we see that the maximum charge ratio $(\frac{q_d}{q_R})^*$ increases with decreasing external electric field and decreasing diameter. Therefore, the charge ratio curve in Figure 14 approaches 1 close to full evaporation.

Figure 14 depicts a comparison of evaporation-driven charge ratio against the dynamic instability threshold induced by the external electric field. Charge ratio $\frac{q_d}{q_R}$ is given as a function of time t for different values of evaporation rate constant

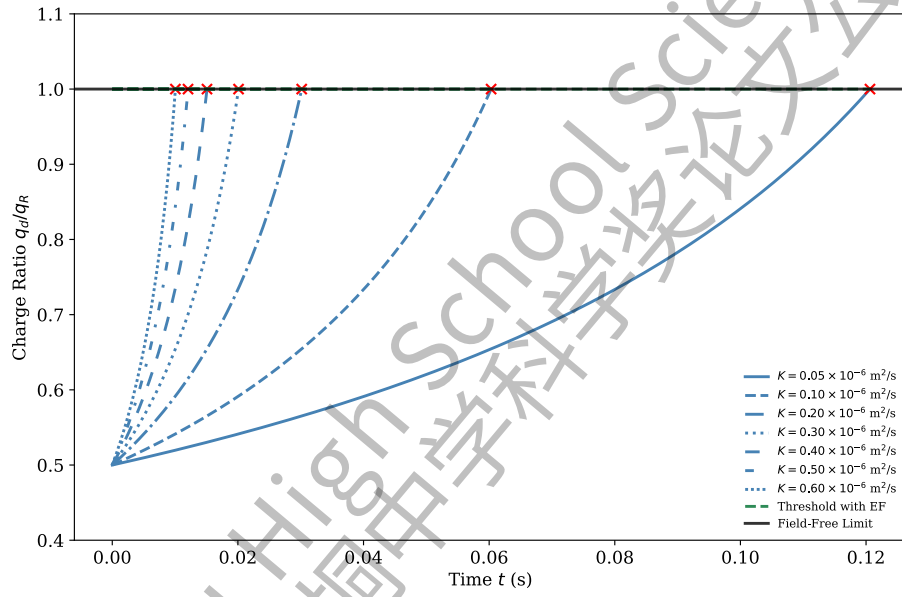


Figure 14: Comparison of evaporation-driven charge ratio against the dynamic instability threshold induced by the external electric field. Charge ratio, $\frac{q_d}{q_R}$, as a function of time, t , for different values of evaporation rate constant, K . Initial diameter, d_0 , is kept constant at 10^{-4}m .

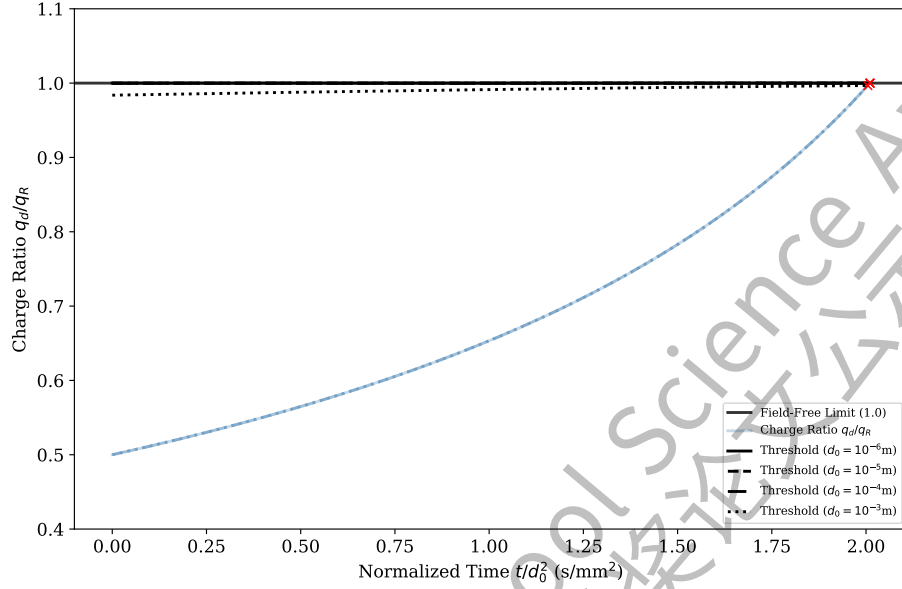


Figure 15: Comparison of evaporation-driven charge ratio against the dynamic instability threshold induced by the external electric field. Charge ratio, $\frac{q_d}{q_R}$, as a function of normalized time, $\frac{t}{d_0^2}$, for different values of initial diameter, d_0 . Evaporation rate constant is kept at $K = 0.3 \times 10^{-6} \text{ m}^2/\text{s}$.

K , and the initial diameter d_0 is kept constant at 10^{-4} m . Each individual charge ratio curve exhibits an increase from 0.5, with higher values of K accelerating the rate of charge accumulation relative to the Rayleigh limit, which eventually causes the breakup to occur at shorter times t . In contrast, instability thresholds induced by external electric field for all values of K remain tightly clustered beneath the field-free Rayleigh limit line $\frac{q_d}{q_R} = 1$. Since d_0 remains constant, the initial electric field E_0 is the same for every case, resulting in an almost identical initial threshold due to the perfect overlapping of instability thresholds for all values of K . Furthermore, because the field effect for a droplet with a diameter of 10^{-4} m is very weak, the threshold approaches 1 almost immediately as the evaporation proceeds, causing the breakups to occur at the classical Rayleigh limit across all evaporation rates.

Figure 15 depicts a comparison of evaporation-driven charge ratio against the dynamic instability threshold induced by the external electric field. Charge ratio $\frac{q_d}{q_R}$ is given as a function of normalized time $\frac{t}{d_0^2}$ for different values of initial diameter d_0 , and the evaporation rate constant is kept at $K = 0.3 \times 10^{-6} \text{ m}^2/\text{s}$. Since the charge ratio is plotted against the normalized time $\frac{t}{d_0^2}$ instead of time t itself, the charge ratio curves for all initial droplet sizes collapse into a single

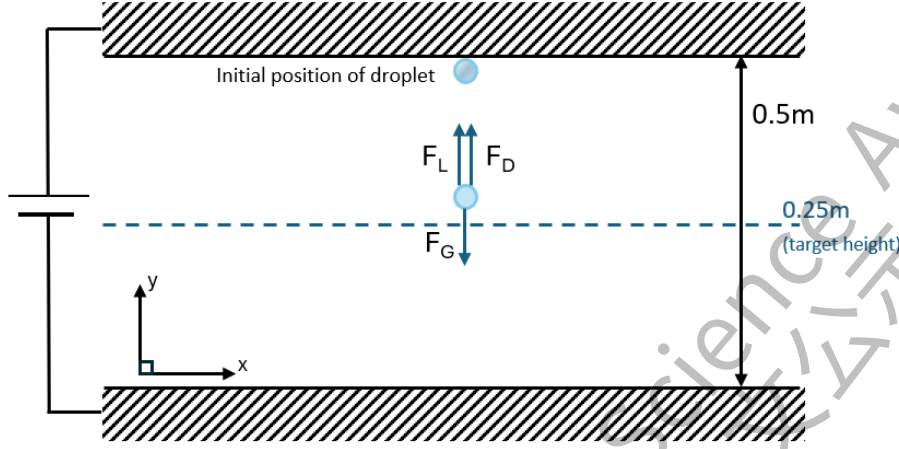


Figure 16: Free-body diagram of investigation sub-case 1.

universal trajectory, rising from an initial value of 0.5 toward 1. Concurrently, the instability thresholds for all values of initial diameters d_0 remain nearly identical to the field-free Rayleigh limit of 1. Nonetheless, a notable deviation occurs only for droplets with larger initial diameter ($d_0 = 10^{-3}\text{m}$, where the threshold starts slightly below 1 at $\frac{t}{d_0^2} = 0$. This downward shift occurs because larger droplets require a strong initial external electric field to balance gravity, which then enhances charge accumulation and lowers the maximum sustainable charge ratio. As the droplet evaporates and shrinks, the required balancing electric field diminishes, causing the instability threshold to rise upward and converge toward 1.

4.4 Stabilization for non-evaporating droplet

4.4.1 Variation of the Initial Velocity

The forces acting on the droplet in sub-case 1 is shown in Figure 16.

The initial velocities needed to stabilize the droplet at a height of 0.25m when released at a height of 0.5m for droplet diameters from 10^{-4} to 10^{-2} meter are shown in Figure 17.

The data reveals a non-linear, stark inverse relationship between the droplet's diameter and the magnitude of the initial velocity required to stabilize it at the target height of 0.25m. As the droplet diameter increases from 10^{-4} to 10^{-2}m , the magnitude of initial velocity decreases drastically from 30.94 m/s to 0.00089 m/s, approaching zero. The phenomenon is governed by the electrostatic equilibrium in Eq.(17), where the upward electric force is balanced by the downward gravitational force, so the drag force provides the only net force in Eq.(1) acting to decelerate the moving droplet:

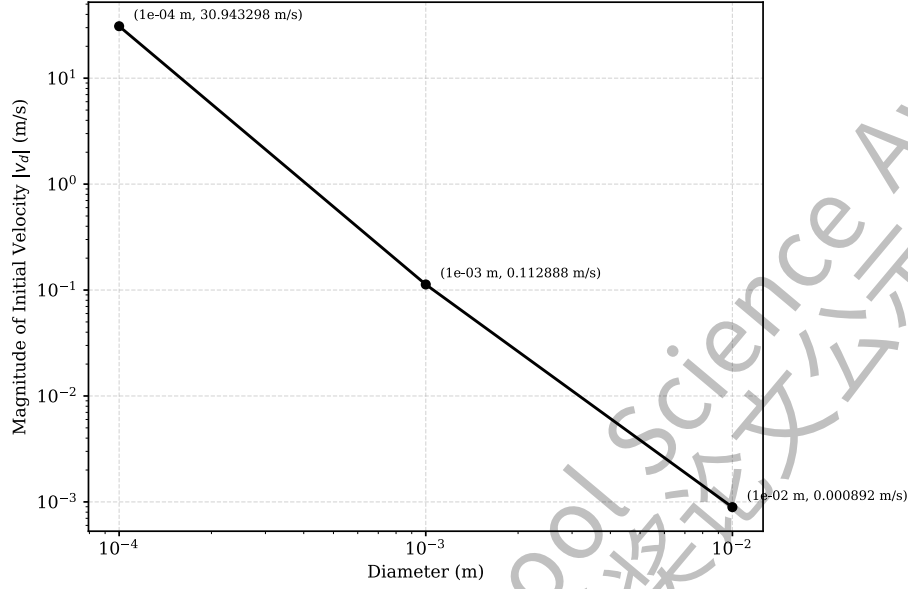


Figure 17: *Initial velocity needed to stabilize the droplet at a height of 0.25m when released at a height of 0.5m as a function of different particle diameters. Both axes in logarithmic scale.*

$$m\vec{a} = \underbrace{\vec{F}_G + \vec{F}_L}_{=0} + \vec{F}_D.$$

Eliminating these two balanced forces gives

$$\vec{a} = \frac{\vec{F}_D}{m}. \quad (39)$$

Substituting the drag force $\vec{F}_D = -3\pi f_{Kn} d \mu_g (\vec{v}_g - \vec{v}_d)$ (for simplicity, Stoke's law is used with a correction factor; however, computations are done with the full drag force) and the mass of the droplet $m = \frac{\pi}{6} d^3 \rho_d$, which ρ_d is the droplet density, we have

$$\vec{a} = -\frac{3\mu f_{Kn} d \pi \vec{v}_d}{\frac{\pi}{6} d^3 \rho_d} = -\frac{18\mu f_{Kn} \vec{v}_d}{\rho_d d^2}, \quad (40)$$

with $\vec{v}_g = 0$ given that a quiescent medium is considered. Thus, for the same magnitude of initial velocity $|\vec{v}_d|$, drag decelerates much more for droplets with smaller diameter due to the inverse relationship with respect to d . As a result, smaller droplets should be injected with a higher magnitude of initial velocity v_d .

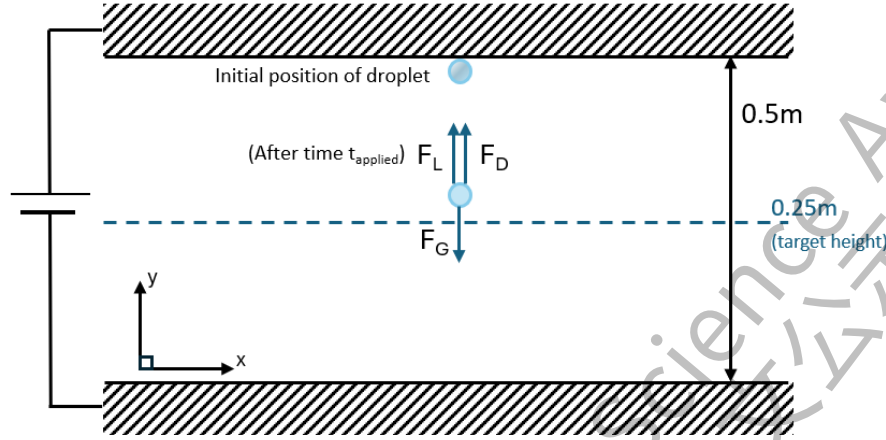


Figure 18: Free-body diagram of investigation sub-case 2.

4.4.2 Variation of the Time of Activation of the Electric field

The forces acting upon the droplet in sub-case 2 is shown in Figure 18.

The times of application of electric field needed to stabilize the droplet at a height of 0.25m when released at a height of 0.5m for droplet diameters from 10^{-5} to 10^{-2} m are shown in Figure 19.

The scatter plot demonstrates a stark, non-linear inverse relationship between the droplet's diameter and the time required to stabilize it at the target height of 0.25m. The time shifts from 83.59 to a mere 0.0001s and approaches zero when droplet diameter increases from 10^{-5} to 10^{-2} m. The behavior of the droplets falls into two distinct phases. In phase 1, gravity brings the particle downwards, while the drag force tends to slow it down. However, with only these forces, the particle continues to fall down. To stop it from falling down, we need to remove the gravitational force, which is done by adding an extra force (electric force) that balances gravity, beginning phase 2. After this force is applied, the only remaining force is drag, which dissipates all the energy until the droplet stabilizes. Since small droplets have almost negligible mass, aerodynamic drag completely dominates them, and this results in a very slow terminal velocity. As the smaller droplets drift downward so slowly, the system waits a longer period of time for them to finally reach near the 0.25m target height before switching on the electric field to stabilize it. This is because the negligible mass means virtually negligible inertia, so the smaller droplets stop almost immediately when the electric field is turned on.

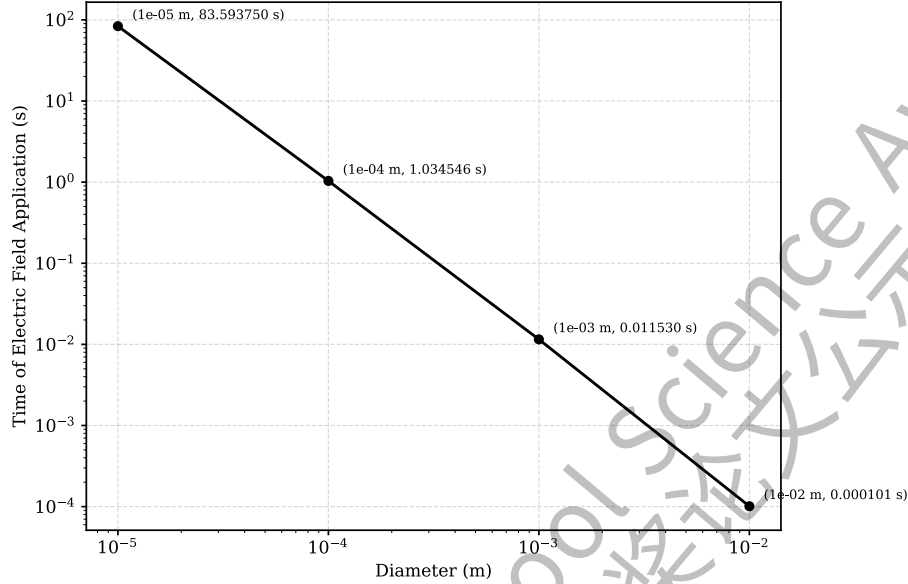


Figure 19: *Times of application of electric field needed to stabilize the droplet at a height of 0.25m when released at a height of 0.5m as a function of different particle diameters. Both axes are in logarithmic scale.*

4.5 Trajectory of evaporating droplets with no stabilization

4.5.1 Variation of the Initial Velocity

To investigate the effect of evaporation on droplet stabilization, a comparison of the motion of a charged water droplet with and without evaporation is done. The droplet is initially released at a height of 0.5m and moves vertically through air under the combined forces of gravity, aerodynamic drag, and an applied electric field. The droplet has an initial diameter of 1.0×10^{-3} m, and its charge is set to 0.5 of the Rayleigh limit. The electric field is applied from the beginning of the simulation and is initially chosen so that the electric force balances the droplet's weight. For the evaporating case, the droplet diameter decreases according to the d^2 -law, with an evaporation rate constant $K = 1.0 \times 10^{-8}$ m²/s, while its charge is kept constant. The graphs of the droplet trajectory, force balance, and vertical velocity are then compared with those of the corresponding non-evaporating case to determine how evaporation influences the droplet's motion.

Figure 20 shows the vertical position y of the droplet as a function of time t for both evaporating and non-evaporating conditions. Both droplets are released at $y = 0.5$ m with the same calibrated initial downward velocity. The marker

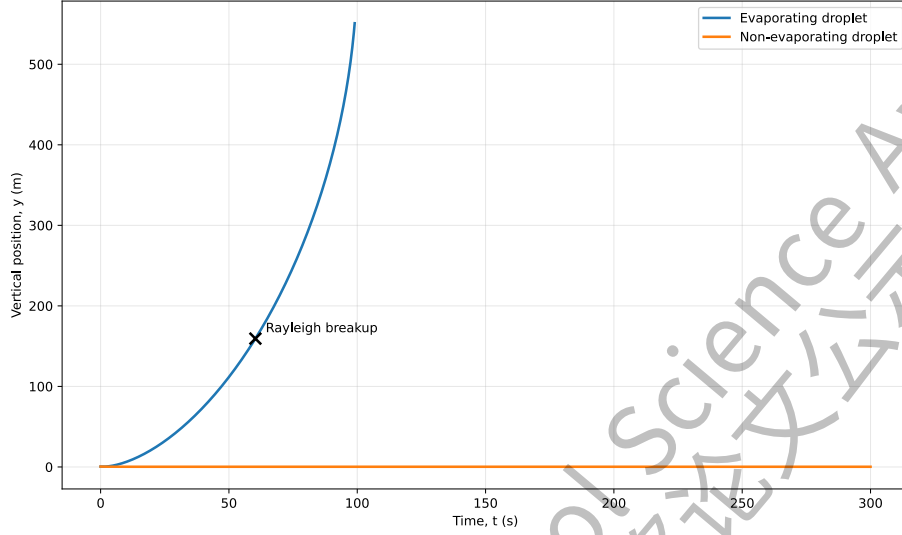


Figure 20: Vertical position, y , of water droplet as a function of time, t , for both evaporating and non-evaporating conditions in the case of varying initial velocity.

on the evaporating curve indicates the time instant of Rayleigh breakup, which was found from Eq.(37) by imposing $\frac{q_d}{q_R} = 1$. The non-evaporating droplet moves downward towards the target position of 0.25m in a short amount of time. In contrast, the evaporating droplet initially moves downward for a very short amount of time but then subsequently slows, stops, reverses the direction and accelerates upwards. This is due to the fact that evaporation reduces the droplet diameter and thus its mass, which then results in the decrease of the gravitational force $F_G = mg$ while electric force $F_E = qE_s$ remains constant because the charge and electric field are fixed. Consequently, the upward electric force dominates the downward gravitational force, producing an increasing upward acceleration. The trajectory therefore changes from decreasing y to increasing y , with the increasingly steep gradient showing the accelerating upward motion.

Figure 21 compares the drag, gravitational and electric forces acting on the droplet as a function of time t for both evaporating and non-evaporating conditions. The markers on the evaporating curves indicate the time instant of Rayleigh breakup, which was found from Eq.(37) by imposing $\frac{q_d}{q_R} = 1$. The electric force remains constant in both cases because the droplet charge and the electric field are unchanged, where $F_E = qE_s$. For the non-evaporating droplet, the gravitational force is also constant because its diameter and mass remain unchanged. The drag force starts at a small, positive initial magnitude driven by its initial injection velocity, but it rapidly decays to zero within the first few

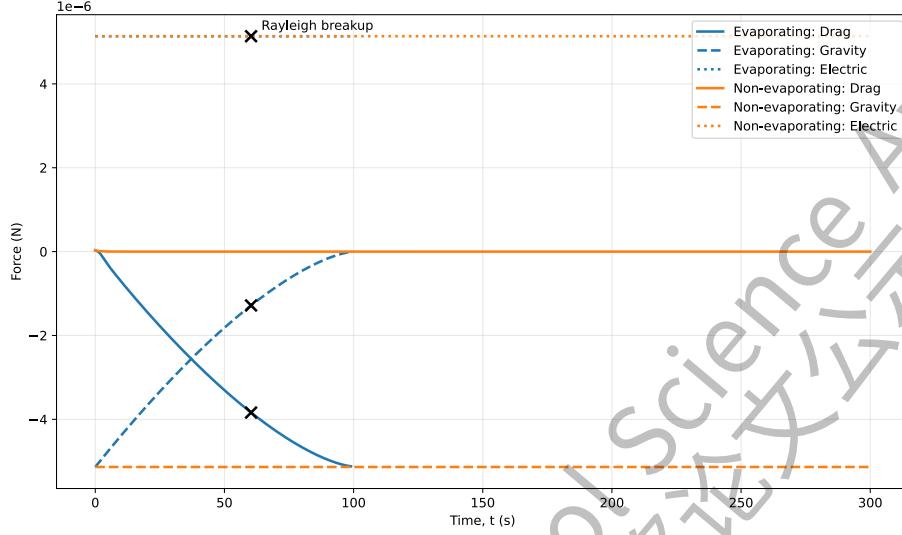


Figure 21: Drag, gravitational, and electric forces acting upon water droplet as a function of time, t , for both evaporating and non-evaporating conditions in the case of varying initial velocity.

seconds and stays zero thereafter. This is due to the loop that drag itself slows the droplet's vertical motion and yet the decreased vertical velocity reduces drag itself. For the evaporating droplet, however, the magnitude of the gravitational force $F_G = mg$ decreases continuously because the mass decreases as the droplet evaporates. Additionally, the drag force is determined by both droplet size and velocity. The reduction in diameter decreases the surface area and therefore reduces drag, while the increasing velocity of the evaporating droplet increases drag because drag depends directly on the square of the relative velocity as shown in Eq.(7). At the end, since the velocity increases at a rate that far outweighs the reduction in droplet size, the magnitude of the drag force increases gradually until complete evaporation occurs.

Figure 22 presents the vertical velocity of the droplet as a function of time t for both evaporating and non-evaporating conditions. The marker on the evaporating curve indicates the time instant of Rayleigh breakup, which was found from Eq.(37) by imposing $\frac{q_d}{q_R} = 1$. For the non-evaporating droplet, the negative initial velocity becomes zero in a short amount of time. In contrast, for the evaporating droplet, as the evaporation progresses, the velocity becomes less negative until it reaches zero, at which point the droplet only momentarily stops. The velocity then becomes positive, indicating that the droplet has reversed direction and is moving upwards. The increasingly steep rise in velocity occurs because the droplet's mass decreases while the electric force remains constant. As mentioned before, the net upward acceleration due to the dominance of the

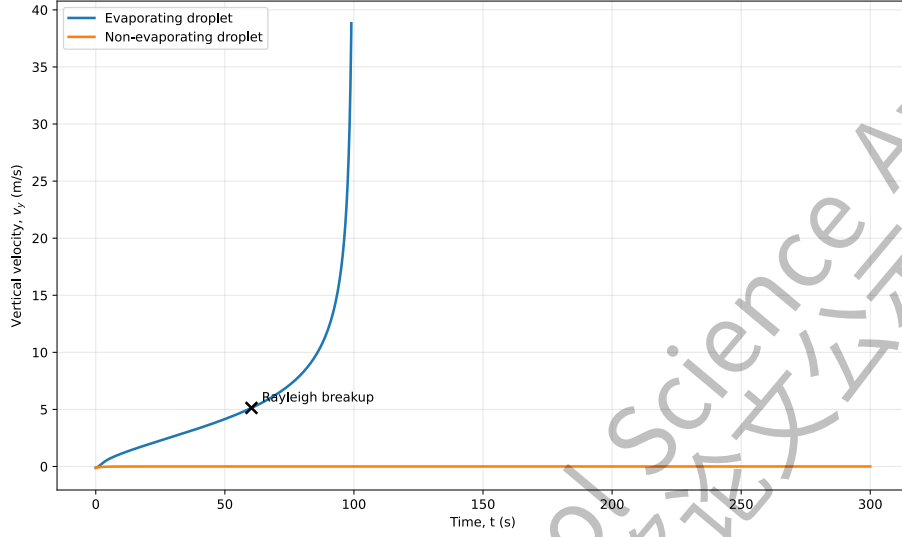


Figure 22: Vertical velocity, v , of water droplet as a function of time, t , for both evaporating and non-evaporating conditions in the case of varying initial velocity.

electric force becomes increasingly significant as evaporation continues. The velocity graph provides direct evidence for the mechanism, where the positive velocity has an increasingly steep gradient that reflects the increase in upward acceleration over time due to mass loss.

4.5.2 Variation of the Time of Activation of the Electric field

Having examined the effect of variation of the initial velocity, we now investigate the effect of varying the time at which the electric field is activated. The trajectory and vertical velocity graphs produced for this case exhibit generally similar trends to those in the first investigated case. The main difference is only a slight change in the timing and magnitude of the droplet's motion caused by a slightly different electric-field activation time, while the overall behavior of the droplet remains unchanged. To avoid largely repeating the conclusions established previously in the first case, the analysis here focuses only on the force graph, where the trend of the electric force is clearly different.

Figure 23 compares the drag, gravitational and electric forces acting on the droplet as a function of time t for both evaporating and non-evaporating conditions. The markers on the evaporating curves indicate the time instant of Rayleigh breakup, which was found from Eq.(37) by imposing $\frac{q_d}{q_R} = 1$. Before the electric field is activated, the electric force is zero, with only drag and gravitational forces acting upon the droplet. However, at the activation time, the electric force abruptly increases from zero to a constant value, appearing as a

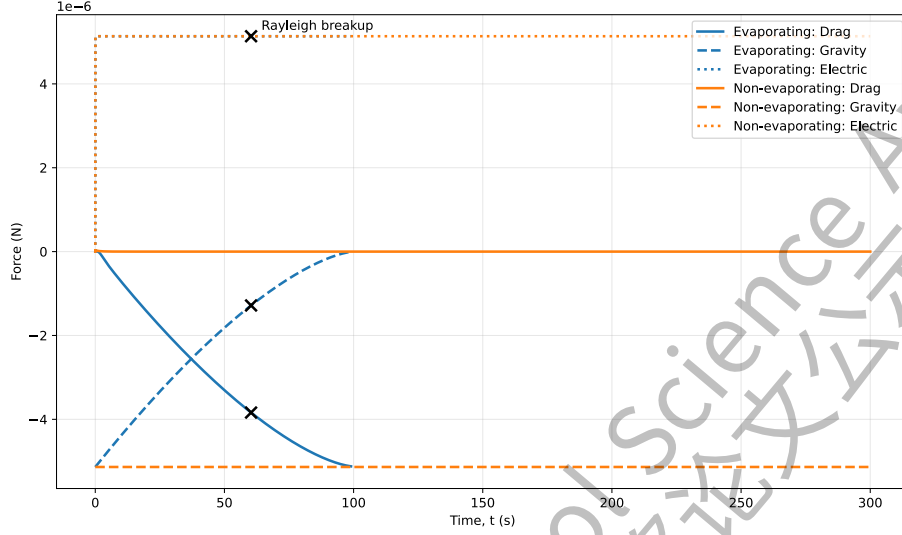


Figure 23: Drag, gravitational, and electric forces acting upon water droplet as a function of time, t , for both evaporating and non-evaporating conditions in the case of variation of time of activation of electric field.

vertical straight line. This occurs because the electric field is applied suddenly at $t = t_e$, so the electric force changes from $F_E = 0$ to $F_E = qE_S$. The electric force then remains constant after activation because both the droplet charge and applied electric field are held constant. Conclusions about the two other forces are the same as those explanations in the previous case.

4.6 Stabilization for evaporating droplet with Rayleigh breakup

Substituting in $y_{target} = 0.25\text{m}$ and $y_0 = 0.5\text{m}$, the generic model for droplet trajectory, Eq.(26), becomes

$$y(t) = 0.25 + 0.25e^{-\frac{t}{\tau}}. \quad (41)$$

The equation of electric field as a function of time, Eq.(29), can thus be expressed as

$$E(t) = \frac{\sqrt{2}}{2d_0^{\frac{3}{2}}\sqrt{\epsilon_0\sigma}} \left[\frac{\rho_d}{6}(d_0^2 - Kt)^{\frac{3}{2}} \left(g + \frac{e^{-\frac{t}{\tau}}}{4\tau^2} \right) - \frac{3\mu_g}{4\tau} \sqrt{d_0^2 - Kte^{-\frac{t}{\tau}}} \left(1 + \frac{1}{6} \left(\frac{\rho_g \sqrt{d_0^2 - Kte^{-\frac{t}{\tau}}}}{4\tau\mu_g} \right)^{\frac{2}{3}} \right) \right]. \quad (42)$$

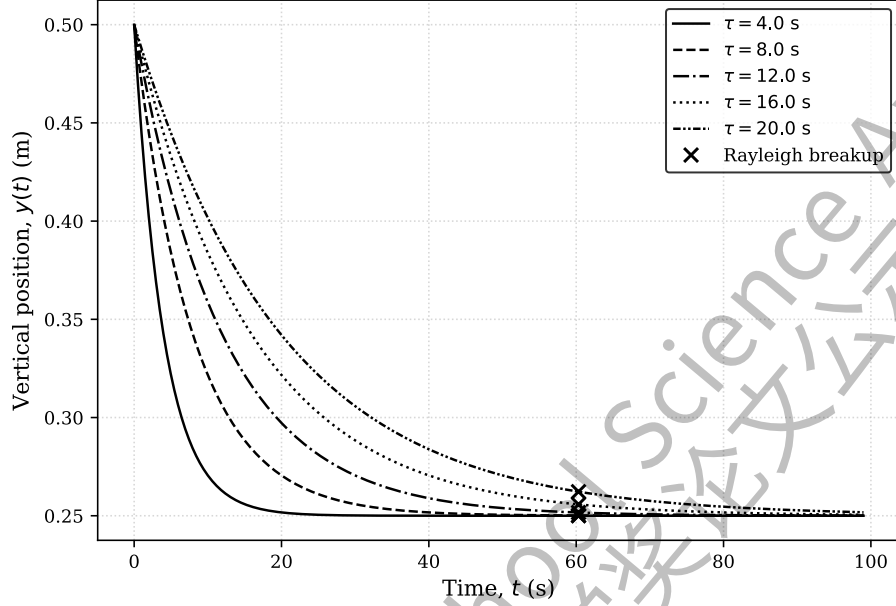


Figure 24: Vertical position, $y(t)$, of water droplet as a function of time, t , for with different values of time constant, τ . Evaporation rate constant, K , is $1.0 \times 10^{-8} \text{m}^2/\text{s}$ and initial droplet diameter, d_0 , is $1.0 \times 10^{-3} \text{m}$. Modeling of droplet trajectory from Eq.(41).

The two equations are now analyzed with different time constants used, as the time constant τ determines how quickly the droplet reaches its stabilization. The five time constants ($\tau = 4\text{s}, 8\text{s}, 12\text{s}, 16\text{s}, 20\text{s}$ in the case of $d_0 = 1.0 \times 10^{-3} \text{m}$ and $K = 1.0 \times 10^{-8} \text{m}^2/\text{s}$) were selected as fractions at or below one-fifth of the total evaporation lifetime ($t_{\text{evap}} = 100\text{s}$), calculated from the d-square law from Eq.(16) by setting $d_{t_{\text{evap}}} = 0$,

$$t_{\text{evap}} = \frac{d_0^2}{K}. \quad (43)$$

Since a charged droplet encounters Rayleigh breakup when charge density exceeds surface tension forces, which in our case $t_{\text{break}} \approx 60.31\text{s}$ assuming initial diameter $d_0 = 1.0 \times 10^{-3} \text{m}$ and evaporation rate constant $K = 1.0 \times 10^{-8} \text{m}^2/\text{s}$, the trajectory must settle the droplet near its target height before the breakup. Restricting time constant $\tau \leq 0.2t_{\text{evap}}$ ensures that the electric field acts quickly enough to stabilize the droplet before the loss of mass triggers the eventual breakup.

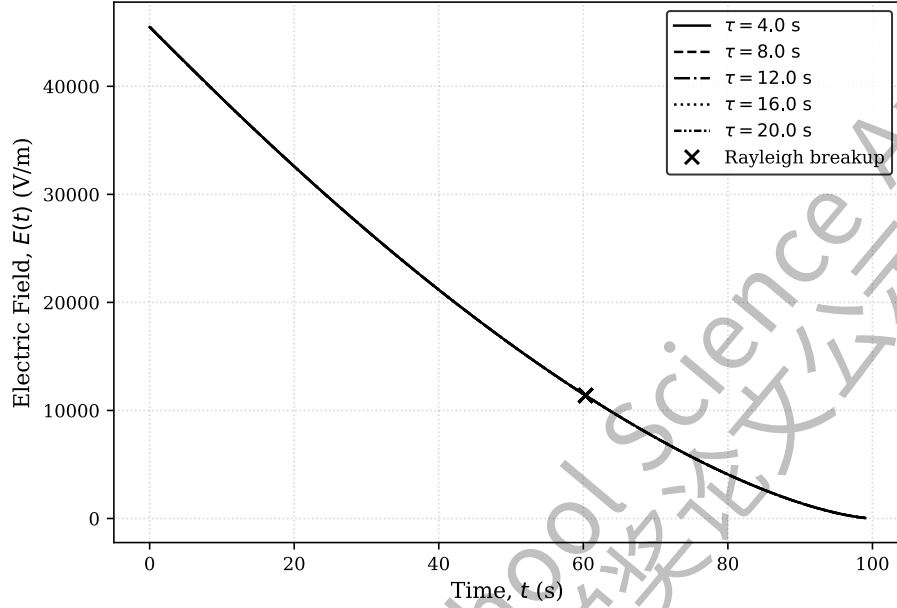


Figure 25: Electric field, $E(t)$, applied as a function of time, t , for different values of time constant, τ . Evaporation rate constant, K , is $1.0 \times 10^{-8} \text{m}^2/\text{s}$ and initial droplet diameter, d_0 , is $1.0 \times 10^{-3} \text{m}$. Modeling of analytical electric field function from Eq.(42).

Figure 24 presents the vertical position $y(t)$ of a water droplet as a function of time t , modeling the trajectory of the droplet from Eq.(41) for different values of the time constant τ . Across all five curves, the vertical position of the droplet exhibits smooth exponential decay, with smaller τ values yielding steeper initial decline rates and reaching the target height faster. This is because τ is the denominator of the negative exponent $-\frac{t}{\tau}$, so a lower value of τ makes this component more negative, forcing the term $e^{-\frac{t}{\tau}}$ to approach zero and thus the entire vertical position $y(t)$ to reach and flatten out at 0.25m much sooner.

Figure 25 shows the electric field $E(t)$ applied as a function of time t , Eq.(42), for different values of the time constant τ . All five curves begin with an intense spike at $E(0)$, followed by a steep, rapid decay where five curves merge into a single overlapping curve. This overlap occurs because the electric field needs to perform two jobs: supporting the droplet's weight against gravity, and supplying a very small extra force to adjust its position toward the target height. Because the electric force needed to support the droplet's weight is dominantly larger than the force needed to move it, varying the time constant τ only changes the electric field magnitude in a negligible amount. As time progresses and the exponential term $e^{-\frac{t}{\tau}}$ approaches zero, the electric field is purely needed to

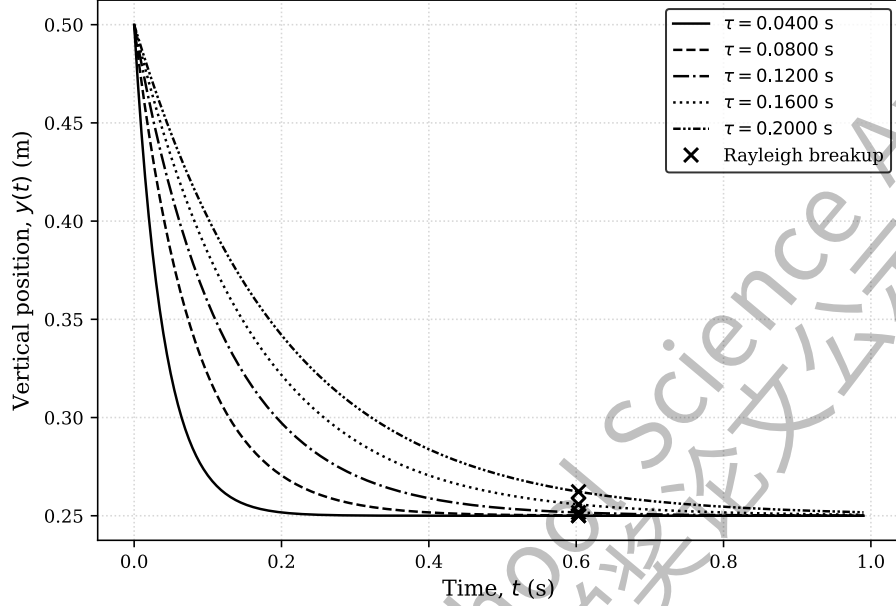


Figure 26: Vertical position, $y(t)$, of water droplet as a function of time, t , for with different values of time constant, τ . Evaporation rate constant, K , is $1.0 \times 10^{-6} \text{m}^2/\text{s}$ and initial droplet diameter, d_0 , is $1.0 \times 10^{-3} \text{m}$. Modeling of droplet trajectory from Eq.(41).

support the droplet's shrinking weight:

$$E(t) \approx \frac{\sqrt{2}\rho_d g}{12d_0^{\frac{3}{2}}\sqrt{\epsilon_0\sigma}}(d_0^2 - Kt)^{\frac{3}{2}}. \quad (44)$$

This leads to a time dependence of the stabilizing electric field, which is expressed through the factor $(d_0^2 - Kt)^{\frac{3}{2}}$ in Eq.(44). As it has negligible dependence on τ , all five curves overlap with each other until Rayleigh breakup occurs.

Now we adjust the evaporation rate constant K to $1.0 \times 10^{-6} \text{m}^2/\text{s}$ and analyze both graphs again. The total evaporation lifetime, the time of Rayleigh breakup, and the five time constants τ are all compressed by two orders of magnitude.

Figure 26 presents the vertical position $y(t)$ of a water droplet as a function of time t , modeling the trajectory of the droplet from Eq.(41) for different values of the time constant τ . Altering the evaporation constant K to $1.0 \times 10^{-6} \text{m}^2/\text{s}$ does not change the shape of the trajectory, except for the compression of the time and time constant values by two orders of magnitude.

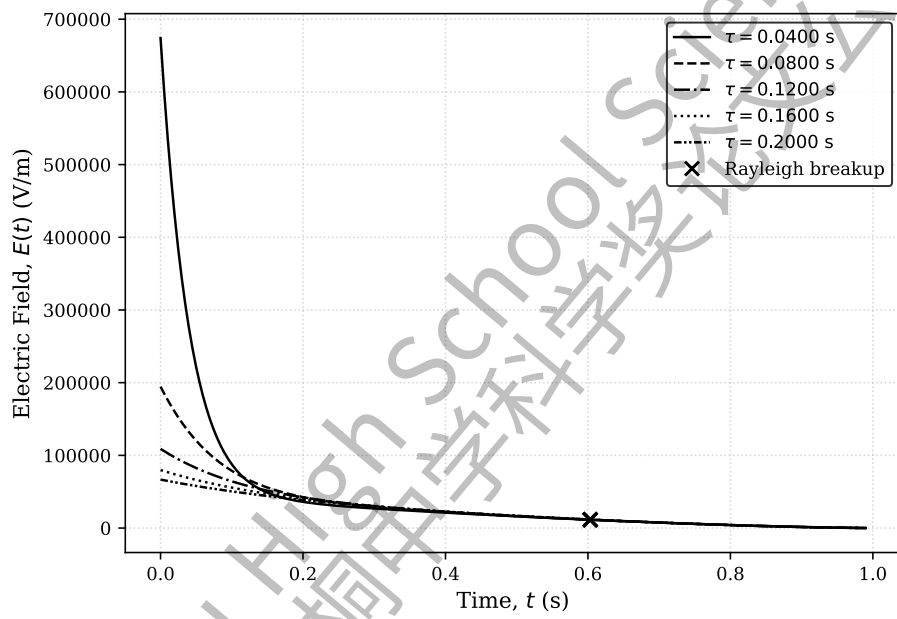


Figure 27: Electric field, $E(t)$, applied as a function of time, t , for different values of time constant, τ . Evaporation rate constant, K , is $1.0 \times 10^{-6} \text{m}^2/\text{s}$ and initial droplet diameter, d_0 , is $1.0 \times 10^{-3} \text{m}$. Modeling of analytical electric field function from Eq.(42).

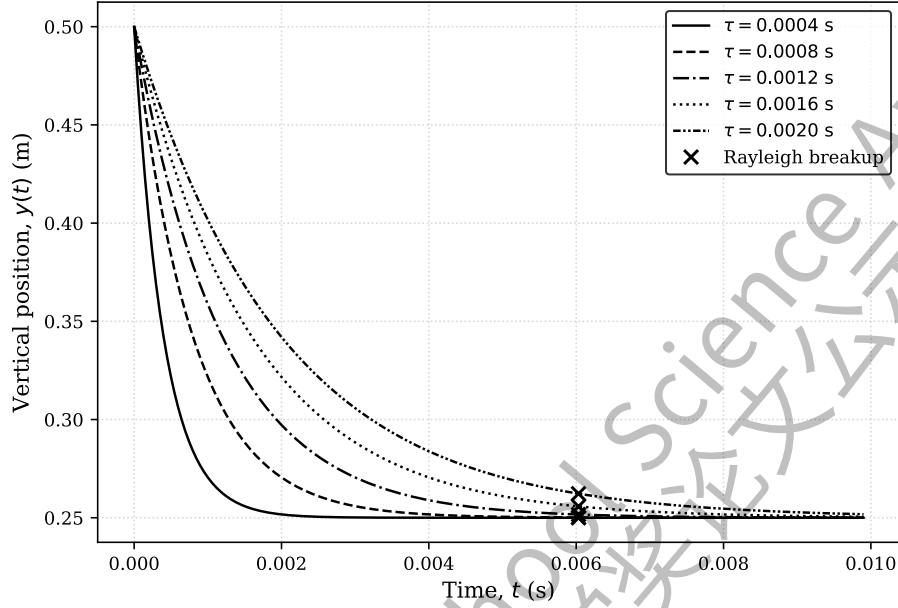


Figure 28: Vertical position, $y(t)$, of water droplet as a function of time, t , for with different values of time constant, τ . Evaporation rate constant, K , is $1.0 \times 10^{-8} \text{m}^2/\text{s}$ and initial droplet diameter, d_0 , is $1.0 \times 10^{-5} \text{m}$. Modeling of droplet trajectory from Eq.(41).

Figure 27 shows the electric field $E(t)$ applied as a function of time t , Eq.(42), for different values of the time constant τ . Unlike the previous slow-evaporation regime, compressing the time and time constant value by two orders of magnitude causes the five curves to diverge noticeably near $t = 0\text{s}$ before merging later. As the terms in Eq.(42) depend inversely on τ and even τ^2 , for small time constants like $\tau = 0.040\text{s}$, the initial acceleration term is about $\frac{0.25}{\tau^2} = 156.25 \text{m/s}^2$, which exceeds gravitational acceleration by sixteen times and thus produces an initial spike at $t = 0\text{s}$. As time progresses, especially beyond $t = 0.3\text{s}$, the exponential term $e^{-\frac{t}{\tau}}$ approaches zero, causing the five curves to collapse onto a single curve.

Now we adjust the initial droplet diameter d_0 to $1.0 \times 10^{-5} \text{m}$ and analyze both graphs again. The total evaporation lifetime, the time of Rayleigh breakup, and the five time constants τ are all compressed by four orders of magnitude.

Figure 28 presents the vertical position $y(t)$ of a water droplet as a function of time t , modeling the trajectory of the droplet from Eq.(41) for different values of the time constant τ . Altering the initial droplet diameter d_0 to $1.0 \times 10^{-5} \text{m}$ does not change the shape of the trajectory, except for the compression of the time and time constant values by four orders of magnitude.

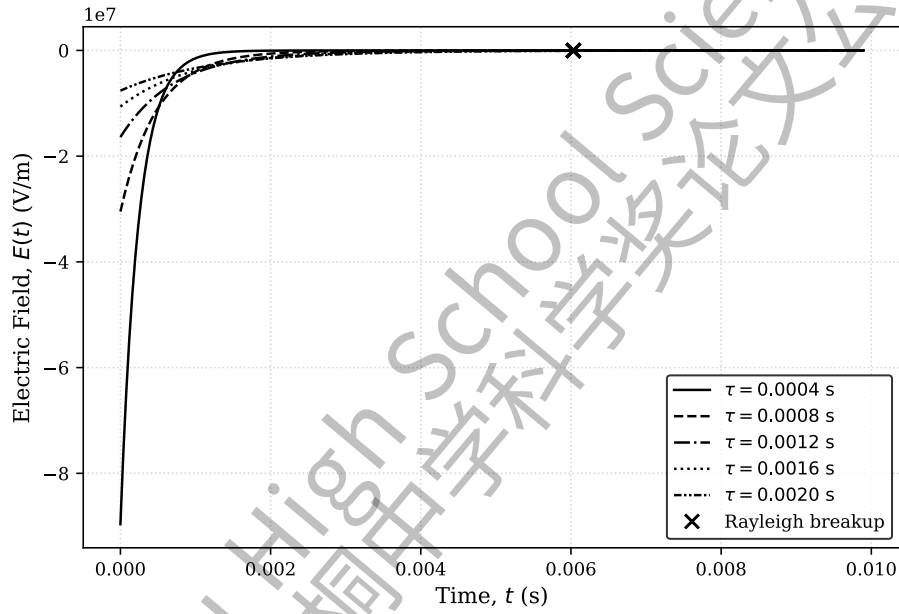


Figure 29: Electric field, $E(t)$, applied as a function of time, t , for different values of time constant, τ . Evaporation rate constant, K , is $1.0 \times 10^{-8} \text{m}^2/\text{s}$ and initial droplet diameter, d_0 , is $1.0 \times 10^{-5} \text{m}$. Modeling of analytical electric field function from Eq.(42).

Figure 29 shows the electric field $E(t)$ applied as a function of time t , Eq.(42), for different values of the time constant τ . For these micro-droplets with diameters $d_0 = 10\mu\text{m}$, the increased surface-area-to-volume ratio causes aerodynamic drag to completely dominate gravitational force at $t = 0\text{s}$. The initial downward speed $-\frac{0.25}{\tau}$ also increases dramatically in magnitude while mass drop by a factor of 1 million, causing the drag to significantly outweigh the gravitational pull. Consequently, the initial electric field is negative for all curves, as the field must invert its direction to pull the droplet downward against strong air resistance. Smaller time constants like $\tau = 0.0004$ result in higher magnitudes of initial speed, causing $E(0)$ to be significantly larger in magnitude and the five curves to diverge at the beginning. As time progresses, especially beyond $t = 0.3\text{ms}$, the exponential term $e^{-\frac{t}{\tau}}$ approaches zero, eliminating the effects of drag and causing the five curves to collapse onto a single curve.

5 Discussion

The results presented in this work illustrate the idea that the trajectory and stability of a charged droplet cannot be simply understood by treating drag, evaporation, and electrostatics in isolation. Section 4.1 established that the Knudsen slip correction becomes significant only for extremely small droplets or reduced ambient pressures where the mean free path of gas becomes large relative to the droplet, a condition of low-pressure electrospray applications but negligible for droplets used later in the trajectory-control cases. Sections 4.2 showed that the stabilizing electric field of an evaporating droplet decays according to the d^2 -law, with the decays collapsing to a single curve when all droplets approach complete evaporation due to the normalization of time by d_0^2 . This scaling reappears more strongly in Section 4.3, where the charge ratio $\frac{q_d}{q_R}$ for droplets of all initial sizes reaches the Rayleigh explosion limit at the same normalized time.

Sections 4.4-4.6 presented results of trajectory control, shifting the focus from passively stabilizing the droplet to actively steering it. A key finding is that the electric field needed to move a droplet to a target height is dominated by the requirement to balance gravity, whereas the magnitude of electric field needed to shape the droplet's trajectory is comparatively small. As a result, the choice of time constant τ actually has a negligible effect on the required field until the droplet becomes very small or the evaporation rate constant becomes very large. In that case the electric field needed to make the droplet follow the trajectory is very high, making active trajectory control very impractical and potentially raising safety concerns such as ionization of the air.

Taken together, the results suggest that evaporation, rather than drag or the electric field individually, is the dominant factor limiting how long a droplet can be actively controlled before Rayleigh breakup occurs.

Future work should largely focus on the following areas. Firstly, this work treats the Rayleigh breakup as the terminal event; therefore, trajectory after the

breakup could be investigated and modeled. Secondly, the droplet is assumed as a point mass with an idealized spherical shape throughout the paper; because static shape deformations significantly change droplet dynamics and natural frequencies [20], the use of three-dimensional codes to resolve the liquid-gas interface and capture deformation effects could be encouraged. Thirdly, the d^2 -law used in this paper assumes a fixed evaporation rate constant K ; a more detailed model coupling evaporation with thermodynamics could be used to investigate the variation of K in time and space. Fourthly, the removal of assumptions and restrictions such as the quiescent-air assumption in Sections 2.4-2.6 and the restriction to a vertical, one-dimensional trajectory would allow the study to be applied to more realistic injection and spray configurations.

6 Conclusions

This work investigated the dynamics and trajectory control of charged droplets under electric fields, particularly focusing on the effects of aerodynamic drag, evaporation, and charged-induced instability on trajectory and stabilization.

The investigation of the effects of slip on the drag force showed that slip becomes increasingly significant as the droplet diameter decreases and the Knudsen number increases, particularly at lower ambient pressures. The correlations of drag illustrated that there are greater relative differences at higher Reynolds numbers, while convergences occur at lower Reynolds numbers. This demonstrates that applying the slip correction is crucial when modeling sufficiently small droplets, where the continuum approximate becomes less accurate.

The investigation of droplet stabilization using electric fields in the absence of evaporation showed that the required initial velocity and time of activation of electric field decrease dramatically as droplet diameter increases. Smaller droplets dominated by aerodynamic drag experience greater deceleration relative to their masses, so they require a larger initial velocity or a later time of activation of electric field to reach the same target height. The results also demonstrate that for droplets with constant mass, the initial velocity and time of application of the electric field are good control parameters to achieve the target height of stabilization once the electric field strength is fixed at the stabilization value.

The investigation and quantification of effects of evaporation on stabilization and trajectory control of droplets demonstrated that evaporation progressively reduces the droplet's mass and thus the gravitational force, yet the electric force remains fixed and unchanged. This eventually causes the droplet to reverse direction and accelerate upward if the strength of the electric field is kept constant. The stabilizing electric field, instead, decreases with evaporation according to the d^2 -law, while the normalization of time by d_0^2 reveals common behavior for electric field decay, especially when all droplets approach complete evaporation, regardless of initial droplet sizes. Stabilization of an evaporating

droplet requires time-control of the electric field strength. This has been shown by imposing a prescribed trajectory of the droplet.

The investigation of the effects of charged-induced instability showed that evaporation causes the charge ratio $\frac{q_d}{q_R}$ to increase until the Rayleigh breakup limit is reached, where higher evaporation rate constant values produce earlier breakup. When time is normalized by d_0^2 , droplets of different initial sizes reach the Rayleigh explosion limit at the same normalized time. The investigation of effects of electric field on the onset of instability demonstrated that the inclusion of an external electric field further lowers the maximum sustainable charge ratio, where the external electric field induces a polarization that triggers the ultimate instability below the classical Rayleigh limit.

Overall, this work provides a fundamental framework for coupling evaporation, aerodynamic drag, electric field control, and charge-induced instability within a single droplet-trajectory model, helping to identify conditions where active electric control remains feasible before eventual Rayleigh breakup. This work also provides a theoretical basis for more realistic trajectory-control studies of charged evaporating droplets in practical applications such as electrospray thrusters in spacecraft propulsion or metal coating.

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Acknowledgements

This research project was initiated to address two key research gaps: the lack of a systematic analysis of slip correction spanning full range of pressures and diameters found in engineering applications and the lack of an investigation of the significance of Rayleigh breakup for active trajectory control under influences of evaporation and droplet sizes. The work thus provides a fundamental framework for coupling evaporation, aerodynamic drag, electric field control, and charge-induced instability within a single droplet-trajectory model, helping to identify conditions where active electric control remains feasible before eventual Rayleigh breakup. The study aims to provide a theoretical basis for more realistic trajectory-control studies of charged evaporating droplets in practical applications such as electrospray thrusters in spacecraft propulsion or metal coating.

The student author, Yu Hin Wong, conducted this project under the academic supervision of Siya Huang, a Physics teacher at the Shenzhen College of International Education. The instructor provided topic selection, theory and computation, data analysis, and thesis writing guidance during weekly supervision meetings. The instructor helped mainly by guiding the student to organize a general structure of the study, explaining the underlying python script of investigations, and proofreading the essay sections written by the student. The instructor did not receive any financial compensation for the guidance. All the methodological equation derivations, literature review, python script writing and debugging, graphical data analysis, and the paper writing were completed by the student alone. The student did not receive any help from other students or adults.

The entire paper was completed by the student alone, and he is the only participant in this competition. Therefore, there is no division of labor. To list the specific responsibilities of the instructor and the student, I divided the research process into five different phases.

During phase 1, topic selection and literature review, the instructor proposed research topics and directions for the student to choose from, provided literature recommendations, and helped the student to refine the final problem statement. The student conducted the literature search mainly using Google Scholar and with some help from AI, completed the corresponding review of the papers, identified research gaps, and eventually came up with a research topic. The initial topic was too general, concentrating on the investigation of effects of evaporation on droplets, and it was narrowed down under the guidance of the instructor by cutting unnecessary information.

During phase 2, methodological computations, the instructor provided theoretical and computational guidance by explaining the equations and theories involved in the study. The student managed to complete the derivations and cite the equations from existing literature, providing computational framework for the results section afterwards. Difficulties often occurred during the computation process, where a variable or constant could easily be overlooked and the simplification processes of equations risk making careless mistakes. The instructor helped identify mistakes within the equations

and told the student to fix them by himself.

During phase 3, investigation design and implementation, the instructor provided guidelines of the setups for the droplet stabilization investigation and explained the core python script responsible for the investigation computation. The student then wrote the python script of the fundamental solver function in Google Colab and began debugging the code. Often the python script included a lot of errors and required frequent debugging, and this was a tedious task. The instructor would then explain the errors so that the student could debug the code. All final code corrections and changes, testing, and script validations were completed independently by the student.

During phase 4, graph creations and data analysis, the instructor offered guidance on the data analysis process and helped determine which graphs to present. The student developed the Python analysis scripts and executed iterative testing, ensuring the graphs produced were accurate and clear. The student then analyzed the graphs one by one following a rigid structure: state clearly what each graph shows, describe the trend shown in each graph, and explain the trend with mathematical or physical reasoning. Difficulties occurred throughout the data analysis process as sometimes the analysis was incomplete or incorrect, or the existing number of graphs was insufficient to interpret a subtopic fully, so more graphs were demanded for the same subtopic. The instructor would read the analyses and check for any computational or explanatory mistakes so that the student could fix them immediately or write more in detail.

During phase 5, paper writing, the instructor provided thesis writing guidance by finalizing the layout and structure, correcting structural flow, and proofreading the drafts. The student drafted all sections of the paper independently, including the Abstract, Introduction, Methods, Investigation Cases, Results, Discussion, Conclusion, and References. Formatting difficulties were often encountered during the paper writing, so the student used standardized LaTeX templates to unify the format. Grammatical errors and the repetition of certain technical terms often existed, and the student worked to improve the writing under the guidance of the instructor.

AI was solely used as an auxiliary tool for literature search leads. AI was not involved in formulating the research question, designing the study, creating core arguments, generating data or results, or writing the main body of the paper. Gemini 3.6 Flash was used in literature search. During literature review, the student often asked AI to provide URL links or titles of academic papers on specific topics, such as the Rayleigh limit. In all instances, the student personally read and verified each suggested paper to evaluate its relevance before deciding whether to cite it or not. This was used about 1-2 times per week throughout the entire progress, totaling approximately 5 hours and averaging about 30 minutes for each usage.